Removing the Constraining Assumption of No Joint Products in Breakeven Analysis

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Abstract

One of the major assumptions of the break-even concept is that there is only one product line. This assumption though, has helped in perfecting the existing mathematical model for finding the breakeven point of a firm and for analyzing the implications of a firm's activity level on its costs and profits, has seriously reduced and hindered considerably the use of the concept in analyzing the economic implications of volume in a multi-product business organization of today's competitive business environment. The need to reverse this trend prompted the researcher to using the Reversed Contribution to Sales Ratio (RCSR) approach to perfect a new formula for analyzing multi products break-even points. This paper uses a fairly-easy-to-understand approach to present the new model. The test result using the product moment correlation coefficient revealed a perfect inverse relationship between the normal Contribution to Sales Ratio (CSR) and the associated RCSR for individual products thus proving that the new model was highly effective and precise in the allocation of the joint-products break-even sales to the individual member products. The implication of this development is that the assumption of 'only one product line' no longer hold in breakeven analysis and hence, recommended for pedagogical purposes.

Introduction

It is not, often that you see a manufacturing or trading firm dealing with only one product line in modern times. This may be so in earlier business ages when competition is minimal and competitors are few. Modern day business requires the spirit of competitiveness which a single product line can no longer satisfy.

In trying to fit into a competitive world, a business has to decide which of its operating costs are relevant and which are not. Fortunately, accounting theorists of the past earlier came up with the concept of marginal costing (fashioned after the economy theory of marginal cost and marginal revenue) and there consequent break-even point theory and analysis. Going by the economists custom of holding other things constant while dealing with a particular variable as in their penchant use of the phrase ***ceteris paribus***, the break even analysis is supposed to deal with only one product line as portrayed by the assumption that: ***there is only on product line*** (*Samuelson, 1980*). This assumption, though, has been used successfully over the years to aid the analysis of operational budgets and short term decision making processes, has seriously limited the use of the break-even analysis in providing solutions to short term decision making problems in our present economic world of multi-product businesses. The idea for this paper arose from the need to offer a better model that can deal holistically with break-even analysis in order to adequately address the problem associated with joint or multi-products especially during operational budget preparations.

Multi – Products Break-Even Point

To calculate the normal break-even point all that is required are:
    (a) The separation of fixed and variable costs from estimated total costs.
    (b) The estimate of the total output capacity
    (c) The variable cost per unit
    (d) The selling price per unit; and
    (e) The total estimated fixed costs

This means that you can determine the single product break-even point in units or in Dollar sales using either:

$$b_1 = f/c \qquad (1) \qquad \text{for break-even qty.}$$
$$b_2 = pf/c \qquad (2) \qquad \text{for break-even sales}$$

Where

b_1 = break even-point in units
b_2 = break-even point in Dollar Sales
f = total fixed costs
c = p - v = contribution per unit
p = Unit Selling Price
v = Unit Variable cost

Or, break-even point can equally be determined using the Cost, Volume, Profit (CVP) charts (Drury, 2003; Horngren, 1982; Lucy, 1999). But for now this can only happen for a single product or overall firm's joint products breakeven point. Until now it is only possible to find a single joint or multiple products break-even point; the problem of finding the break-even point for the individual product members of the joint group becomes another challenge which the existing model has been unable to meet. A recent mathematical experiment by the author of this paper offered a solution to this age long problem using a ***Reversed Contribution to Sales Ratio*** (**RCSR**) approach blended with a perfected algebraic interpolation formula.

The reversed contribution to sales ratio is a figure derived from an attempt to fashion out a joint products breakeven point sharing ratio between the individual products making up the group using the inverse value of each product's normal ***contribution to sales ratio*** (CSR).

The new model works by first analyzing the budgeted operations data into:

(a) *Total Budgeted Sales* (Turnover Estimate);
(b) Total Budgeted profit;
(c) *Total fixed Costs*;
(d) *Contribution per product per unit* (this can be easily derived from subtracting the unit variable cost per product from the unit selling price: **p** - **v** as defined earlier); and
(e) *Contribution to sales ratio* - using the total sales per product to divide the total contribution per product line (Omolehinwa, 1991).

With the above data ready, we can now proceed to make the attendant theoretical assumptions and derive the new mathematical model as follows:

Let   c = total contribution
         w = original contribution to Sales Ratio (CSR) i.e. w = contribution / sales
         d = proportional CSR
         e = reverse value
         y = Reversed Contribution to Sales Ratio (RCSR)

such that:

$d_t = 100 (w_t / \sum_{t=1}^{n} w)$ (3)

$e_t = 100 - d_t$ (4)

$y_t = 100 (e_t / \sum_{t=1}^{n} e)$ (5)

where:
         c = contribution per product
         n = number of products
         t = n^{th} product

Carrying our assumptions further,
     Let   b = Joint products BEP in Dollar Sales
              s = Total sales (all products)
              f = total fixed costs

Such that:

b = fs/c   (6)

$b_{pt} = (y_t b)/100$ (7)

Where
         b_{pt} = individual break-even sales for the t product

The proof of the pudding

To see how the model works, we shall use the following case data to do the analysis:

Malaika Ltd is engaged in the manufacture and sale of 5 products namely A, B, C, D, and E. The following data relate to its proposed budget for the first quarter of 2012:

(i)   Total fixed costs estimate = $148,000
(ii) Other operational data are tabulated as follows:

Table 1: Malaika Ltd's Operational Budget Data – First Quarter 2012

Product	Sales	Variable Costs	USP	UVC
A	250,000	200,000	25	20
B	510,000	450,000	34	30
C	560,000	440,000	28	22
D	280,000	250,000	28	25
E	240,000	180,000	20	15

Required - Analyze the given data above with a view to presenting:

(i)   The over-all break-even point in Dollar sales for the firm; and
(ii) The individual products breakeven point -
        (a) In Dollar sales
        (b) In units
(iii) The over-all margin of safety and the individual products margin of safety

Solution

The required steps in solving this problem are to do an extended analysis of the data in a tabular format using the formulae earlier derived as in table 2 below. The primary procedure is:

Step 1 – Compute the overall contribution for the firm;
Step 2 – Compute the contribution per product line
Step 3 – Compute the Contribution to Sales Ratio (CSR)
Step 4 – Compute the proportional CSR using the formula as per equation (3)
Step 5 – Compute the Reverse Value (RV) of the CSR using equation (4)
Step 6 – Compute the Reversed Contribution to Sales Ratio (RCSR) using equation (5) formula.
Step 7 – Compute the joint products breakeven point in Dollar sales using equation (6) formula.
Step 8 – Apportion the joint products breakeven point in Dollar sales computed in step 7 to the individual product line using the formula in equation (7).
Step 9 – Divide the value computed in step 8 by the product unit selling price (USP) to get the breakeven point in quantity of products.
Note: All the nine steps above can easily be presented with the use a table as when doing a statistical analysis. Table 2 below captured all the calculations from step 1 to step 6. Steps 7 to 9 are presented on table 3.

Table 2: Computation Format for Reversed Contribution/Sales Ratio (RCSR)

Product	Sales	Variable Costs	Contribution	CSR (w)	PCSR (d)	RV (e)	RCSR (y)
A	250,000	200,000	50,000	20.0	22.5	77.5	19.4
B	510,000	450,000	60,000	11.76	13.2	86.8	21.7
C	560,000	440,000	120,000	21.43	24.1	75.9	19.0
D	280,000	250,000	30,000	10.71	12.0	88.0	22.0
E	240,000	180,000	60,000	25.0	28.2	71.8	17.9
Total ($\sum$)	1,840,000	1,520,000	320,000	88.9	100.0	400.0	100.0

Having obtained the values of *y* as in the above table, we now calculate the joint products break-even sale in Dollar as follows:

b = (148,000 x 1,840,000) / 320,000
    = 851, 000 in Dollar sales

To apportion this figure to the joint products on individual products basis, we use the value of *y* from the above table:

Table 3: Computation of Individual Product's Breakeven Points

Product	y Value	Joint BEP	Product BEP	Product USP	BEP in units
	(g)	(h)	(i)	(j)	(k)
	(derived)	(derived)	(gh/100)	(given)	(i/j)
A	19.4	851,000	165,094	25	6,604
B	21.7	851,000	184,667	34	5,432
C	19.0	851,000	161,690	28	5,775
D	22.0	851,000	187,220	28	6,687
E	17.9	851,000	152,329	20	7,617

Margin of Safety on BEP Analysis

The margin of safety is the distance between the breakeven point and the budgeted achievable capacity. It may be represented in Dollar of sales, units of sales or in percentage of the budgeted output. The margin of safety measures the extent to which a product's comfort zone can be stretched before the budgeted estimate is exhausted.

Table 4: Computation of Margin of Safety

Product	Sales	BEP	MOS	% of MOS
Joint	1,840,000	851,000	989,000	53.75%
A	250,000	165,094	84,906	33.96%
B	510,000	184,667	325,333	63.79%
C	560,000	161,690	398,310	71.13%
D	280,000	187,220	92,780	33.14%
E	240,000	152,329	87,671	36.53%

Comparative Tests

Though, the knowledge of the margin of safety on each product is a useful guide, we are more interested on how the individual product's breakeven point relates to the overall budgeted sales figures. This can be done by dividing each products breakeven point in Dollar sales by the total budgeted sales as we have done in table 5 below:

Table 5: Computation of Ratio of Product BEP to Total Sales

Product	Sales	BEP	% of BEP
Joint	1,840,000	851,000	46.25%
A	250,000	165,094	8.97%
B	510,000	184,667	10.04%
C	560,000	161,690	8.79%
D	280,000	187,220	10.17%
E	240,000	152,329	8.28%

Please note that the joint ratio of 46.25% is also equal to the summation of the individual products' ratios.
To find a sufficient measure of association, we compare the original products CSR with the percentages derived above on one hand and the RCSR on the other hand using the product moment coefficient correlation formula and the multiple regression analysis as shown below:

Table 6: Multiple Variables Regression Analysis

Product	BEP/Sales Ratio	Product CSR	Reversed CSR
A	8.97	20	19.4
B	10.04	11.76	21.7
C	8.79	21.43	19
D	10.17	10.71	22
E	8.28	25	17.9

Regression Line > $Y = 0.18584a - 0'00198x_1 + 0.45497x_2$

$R^2 = 0.99998196$

Joint Correlation Coefficient = 0.999999098

TEST OF SIGNIFICANCE OF R^2

Calculated F (two-tailed test) = 55426.67406337
Calculated F (one-tailed test) = 110853.34812674

Table 7: Correlation Coefficient Analysis (BEP Ratio and Product CSR)

Product	BEP-Ratio	CSR
A	8.97 20	
B	10.04	11.76
C	8.79	21.43
D	10.17	10.71
E	8.28	25

The Coefficient of Correlation is: **-1.00**
This coefficient indicates that there is ***Perfect Negative correlation*** between the individual product's breakeven point ratios and the normal Contribution to Sales ratios of the individual products.

Table 8: Correlation Coefficient Analysis (BEP Ratio and Reversed CSR)

Product	BEP-Ratio	RCSR
A	8.97	19.4
B	10.04	21.7
C	8.79	19
D	10.17	22
E	8.28	17.9

The Coefficient of Correlation is: **1.00**
This coefficient indicates that there is ***Perfect Positive correlation*** between the individual product's breakeven point ratios and the normal Contribution to Sales ratios of the individual products.

Discussions

The tabulation in table 3 shows the individual product break-even point both in Dollar sales as well as in quantity. We observed from the question that no information is given about how the fixed costs relate to the individual products. However, given the relationship that we have been able to establish, it is easy to pro-rate the fixed cost using the computed break-even points.

One may want to ask here ***why use the reverse CSR approach?***

To answer this question, we shall first of all look at how break-even point is determined ordinarily. It is evident from the solutions given to many cases of single product CVP analysis that ***the slimmer the contribution margin***, ***the higher the break-even point*** and vice versa. This simply means that you require larger volume of activity to be able to recover total cost when your product's contribution margin is slim than when it is within comfortable level. Many high competitive products are noted for having slim contribution margin owing to the fear of pricing one's products out of the market. But products in a monopolistic or oligopolistic market (from experience) usually carry high contribution margin.

Relating the above analogy to the use of the reverse CSR in computing individual product's BEP in a multi-product setting points to the fact that ***those products with lower contribution margin must have contributed more to the derived joint break-even position*** for the company than those that have higher margins (Moore, 1998; Morse & Zimmerman, 1997; Solomon, 2001). As a result, the effect of the contribution margin for each product will be inversely proportional to the over-all (joint) break-even point in relation to the original contribution to sales ratio. Owing to the fact that this position has never been considered in the past it became almost impossible to ascertain individual product's break-even point using a computed joint or multi-products break-even point.

Looking at the statistical test of significance of the breakeven points computed using the model, it will seem obvious that a perfect solution to the joint products breakeven analysis has been found. With a perfectly negative correlation co-efficient of -1, it is evident that the computed break-even point relate perfectly to the individual products BEP using the joint BEP figure as a base. From a test of multi-variable regression analysis using the computed individual product's breakeven to sales ratio as the dependent variable and the CSR and the RCSR as the independent variables, it is evident from the coefficient of multiple determination (R^2) of 0.99998196 and the high **F** ratio that the CSR and the RCSR are two sides of the same coin (Hillier & Lieberman, 2007). That being the case, there is no gain saying the fact that RCSR is the true key to the determination of individual product's breakeven point in a multi-product breakeven analysis scenario.

Conclusion /Recommendation

Having perused through many analytical considerations of the break-even concept, it is pertinent that a modern and more scientific approach be adopted in unknotting thorny issues found in simple theories that apparently appear to be previously unsolvable. The Reversed Contribution to Sales Ratio (RCSR) approach introduced and adopted in this paper is a milestone in resolving one of such logjam in break-even analysis. With this method, the assumption of "***only one single products line***" no longer holds and should therefore, be expunged from the break-even analysis assumptions.

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