Determination of Dependance of Radian Frequence Impact of its Own Longitudinal Oscillation of Vehicle Trailor Combination on Power Intensity in Joint and on Trailor Acceleration

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Abstract

The paper deals with Radian Frequency Impact of its Own Longitudinal Oscillation of Vehicle trailer combination on Power Intensity and on Trailer´s Acceleration while starting. It shows relation between the spring´s biggest deformation in joint and radian frequency of its own longitudinal oscillation and some other performance. This relation is later used for determination of optimal spring firmness in joint. This method is being compared with spring firmness in joint.

Key words: Radian frequency, acceleration, own longitudinal oscillation, spring firmness, spring preload, deformation, joint stressing

1ST Trailer acceleration while vehicle trailer combination starting.

The highest trailer acceleration while starting the vehicle trailer combination is given by following relation:

$$a'_{\max} = \frac{c}{m'}\bar{x}_{\max} - g_{\varphi} \quad [\mathrm{m\ .\ s^{-2}}] \qquad (1)$$

while:

c spring firmness in joint [N . m^{-1}],
m' trailer´s weight [kg],
$\bar{x}_{\max}$ the biggest joint deformation, $\bar{x}$ vector expressing,
g gravitational acceleration [9, 806 65 N],
φ factor of roadway resistance, given by relation φ = sin α + f cos α (2)
α roadway slope angle [°],
f factor of wheel road resistance.

It is valid to use the following formula: for the biggest strength in trailers joint with the towing car $F'_{\max} = c\bar{x}_{\max}$ (3)

Both these values $a'_{\max}$, $F'_{\max}$ are thus the biggest deformation function in joint $\bar{x}_{\max}$. This biggest deformation is function of variables´ set

$$\bar{x}_{\max} = f\left(c, m, m'\bar{C}, \xi, \bar{x}_0\right) \qquad (4)$$

while:

m weight of towing car,
$\bar{C}$ increasing driving power speed in regard to weight unit of towing,
ζ allowance in trailer´s joint with towing ca,
$\bar{x}_0$ spring preload in joint.

It is possible to simplify relation $\bar{x}_{\max}$ on six variables by summarizing first three variables into radian frequency of its own longitudinal oscillation of vehicle combination

$$\omega_x = \sqrt{\frac{c}{\nu\, m}} \tag{5}$$

while:

$\nu = \dfrac{m'}{m + m'}$ is proportionate trailer´s weight.

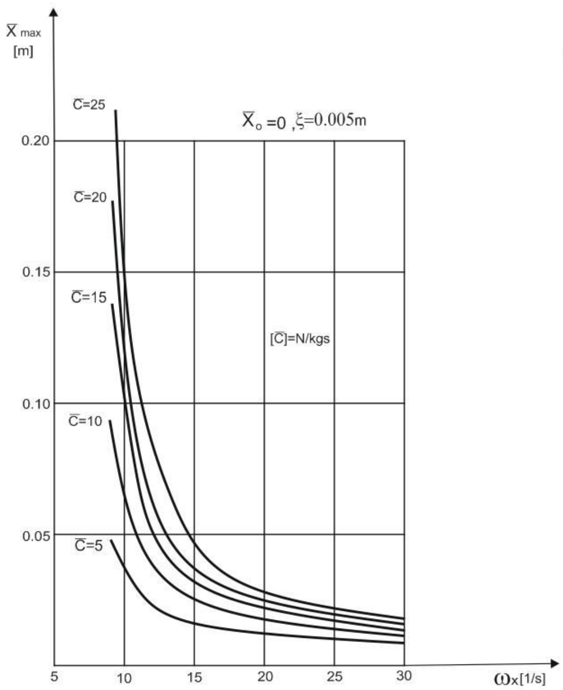
It turned out that this taken relation

$$\bar{x}_{\max} = f\left(\omega_x, \bar{C}, \xi, \bar{x}_0\right) \tag{6}$$

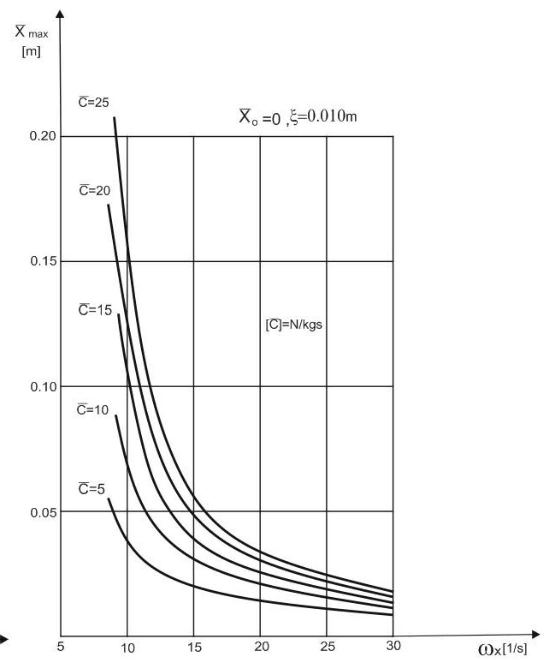
is possible to express by graph system

$$\bar{x}_{\max} = f\left(\omega_x, \bar{C}\right) \tag{7}$$

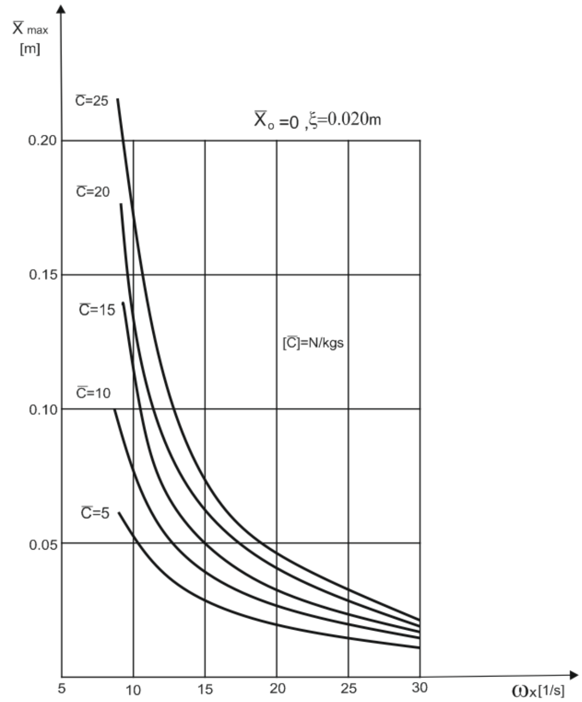
For various values $\bar{x}_0$ and ζ. Calculation results are represented through graphical relations in graphs 1-8.



Graph 1 Relation $\bar{x}_{\max} = f\left(\omega_x, \bar{C}\right)$ for $\bar{x}_0 = 0, \quad \xi = 0{,}005m$

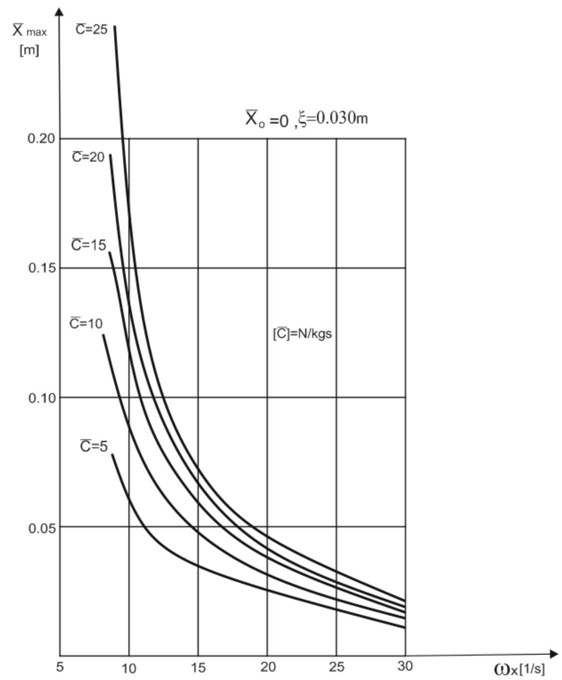


Graph 2 Relation $\bar{x}_{\max} = f\left(\omega_x, \bar{C}\right)$ for $\bar{x}_0 = 0, \quad \xi = 0{,}010m$



Graph 3 Relation $\bar{x}_{\max} = f\left(\omega_x, \bar{C}\right)$

for $\bar{x}_0 = 0, \quad \xi = 0{,}020m$



Graph 4 Relation $\bar{x}_{\max} = f\left(\omega_x, \bar{C}\right)$

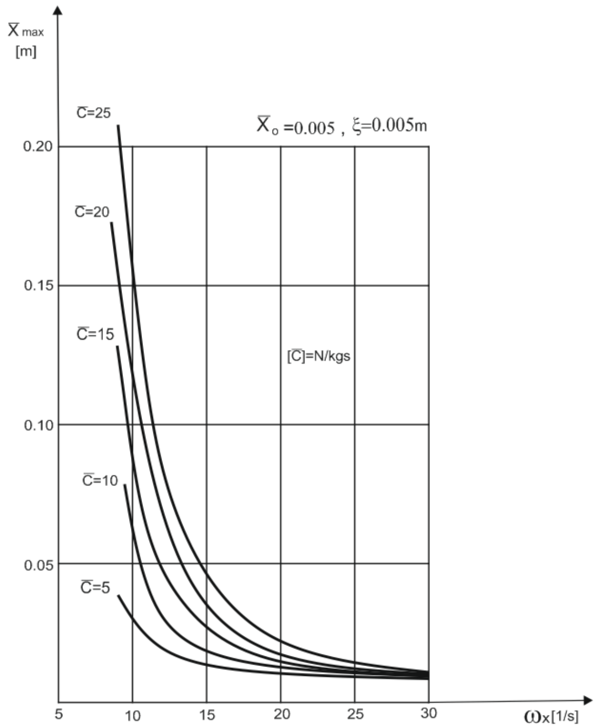
for $\bar{x}_0 = 0, \quad \xi = 0{,}030m$

Comparison of corresponding graphs for various values $\bar{x}_0$ confirms previous conclusion [3] regarding little influence of preload $\bar{x}_0$ to biggest spring deformation size and biggest trailers acceleration. Other three values mainly, ω_x and $\bar{C}$ have rather big influence. Sensitivity of biggest deformation $\bar{x}_{\max}$ into change ω_x is large mainly for little values of radian frequency ω_x.

As it results from the relation (5), radian frequency ω_x is function of three values c, m, m', which is possible to express after application of proportionate trailer´s weight υ, by only two values c and υm. Thus it would be beneficial to investigate, how the biggest strength in joint is varying $F'_{\max}$ and biggest trailers acceleration $a'_{\max}$ if it varies $\omega_{x\ \text{following}}$ the change of one or the other of these values / following the change c or υ m).

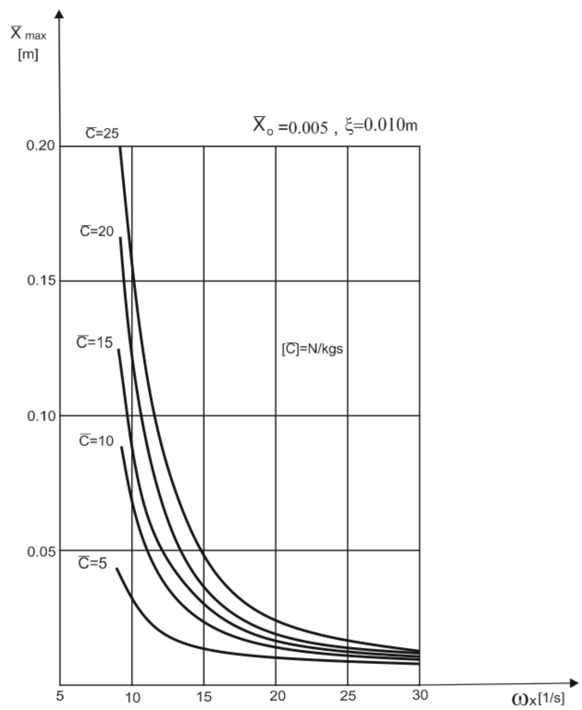
By graphs 5-8 and formula (3) following relation has been stated $F'_{\max} = f\left(\omega_x, \xi\right)$ while υm = 2 000 kg for $\bar{C} = 5N.kg^{-1}.s^{-1}$ (graph 9) and for $\bar{C} = 25N.kg^{-1}.s^{-1}$ (graph 10).

From graphs9 and 10 it is obvious, that while υm = constant factor have curves $F''_{\max} = f\left(\omega_x, \xi\right)$ minimum, that with growing allowance ζ turns to lower values ω_x. Allowance influence ζ on power dimension $F'_{\max}$ is relatively big. Relation of the biggest acceleration of trailer $a'_{\max} = f\left(\omega_x, \xi\right)$ while
υm = constant factor., has similar development as function $F'_{\max} = f\left(\omega_x, \xi\right)$, because following the formula (1) the biggest acceleration of trailer mainly depends on formula $\frac{c}{m'}\bar{x}_{\max}$, which in accordance with relation (3) can be written as follow $\frac{F'_{\max}}{m'}$.



Graph 5 Relation $\bar{x}_{\max} = f\left(\omega_x, \bar{C}\right)$

pre $\bar{x}_0 = 0{,}005m \quad \xi = 0{,}005m$



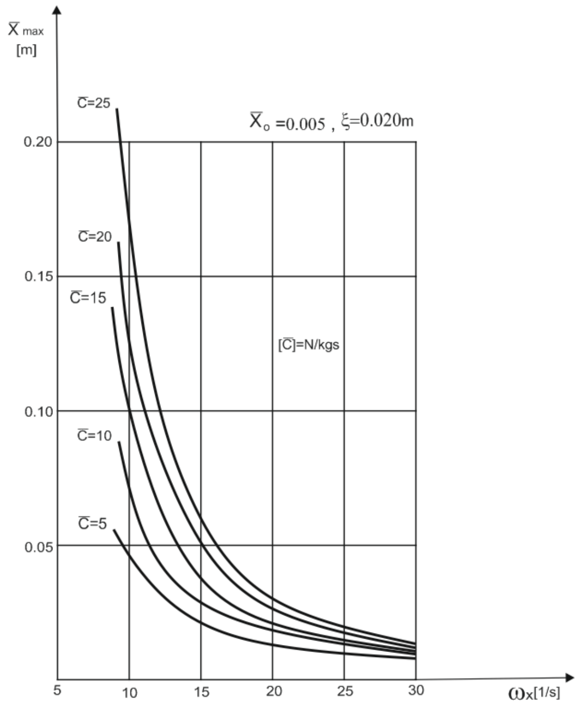
Graph 6 Relation $\bar{x}_{\max} = f\left(\omega_x, \bar{C}\right)$

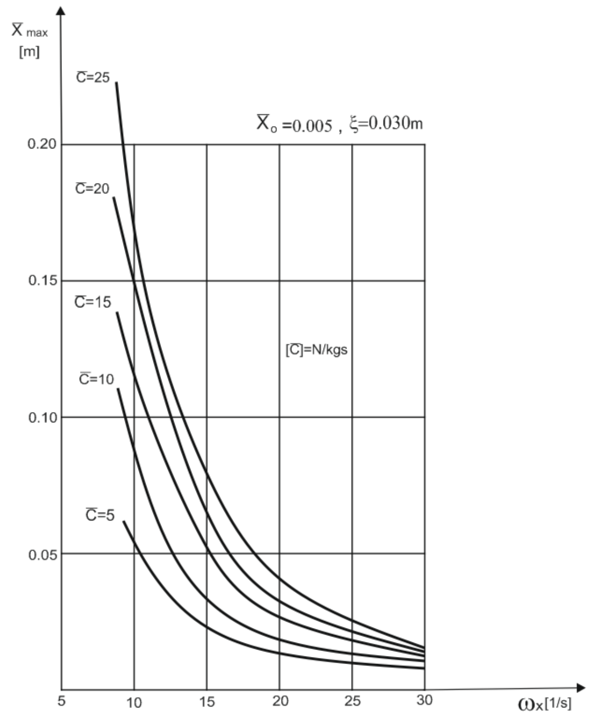
for $\bar{x}_0 = 0{,}005m \quad \xi = 0{,}010m$

Accordingly this function has (while m' = constant factor) minimums under the same values ω_x as function $F'_{\max} = f\left(\omega_x, \xi\right)$. This implies, i tis possible to find such radian frequency ω_x, under which $F'_{\max}$ and $a'_{\max}$ are minimal.

On the contrary if the radian frequency ω_x changes due to modification of term υm, while constant spring rate c = constant factor., have according relations (1) and (3) relations $F'_{\max} = f\left(\omega_x, \xi\right)$, $\frac{c}{m'}\bar{x}_{\max} = f\left(\omega_x, \xi\right)$ while c = constant factor., while their determination was c = 800 000N/, $\bar{C} = 25N.kg^{-1}.s^{-1}$, m = 5 000 kg, $\bar{x}_0 = 0{,}005\,m$.

With growing radian frequency ω_x while (c = constant factor) thus declines the biggest strength in joint $F'_{\max}$ and growths term size $\frac{c}{m'}\bar{x}_{\max}$ and so does the biggest trailer ´s acceleration $a'_{\max}$. It implies, it is not favorable to have υ too small (i.e. big ω_x), either too big (i.e. small ω_x). Relative trailer´s weight used to mainly be between υ = 0,25 – 0,5, which in our case correspond with ω_x = 18 – 25,8 s^{-1}. This range υ really covers medium part of graphs, where there are not too big strengths in joint nor is big trailer´s acceleration.





Graph 7 Relation $\overline{x}_{\max} = f\left(\omega_x, \overline{C}\right)$
for $\overline{x}_0 = 0{,}005m \quad \xi = 0{,}020m$

Graph 8 Relation $\overline{x}_{\max} = f\left(\omega_x, \overline{C}\right)$
for $\overline{x}_0 = 0{,}005m \quad \xi = 0{,}030m$

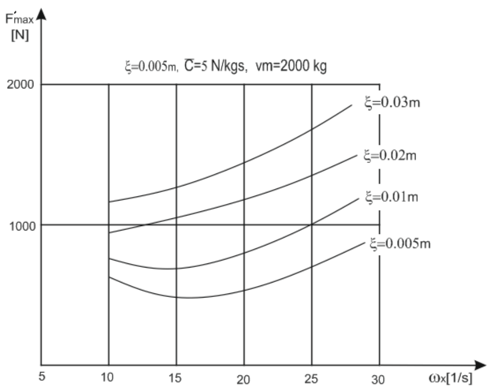
Hereinafter we demonstrate, what shape $F'_{\max} = f\left(\omega_x\right)$ relation has while υm = constant factor, if we recalculate that for several various values υm.

Following values were selected υm = 1 000kg, 2 000kg and 3 000kg while $\overline{C}$ = 25 N . kg^{-1}. s^{-1}, m = 5 000 kg and $\overline{x}_0$ = 0,005 m. There are curves on graph 11 $F'_{\max}$ that are specified for allowances in joint ζ = 0,01 m and on graph 12 for ζ = 0,02 m. From both expressed graphs it is visible that curves $F'_{\max} = f\left(\omega_x\right)$ while υm = constant factor have (while certain value $\overline{C}, \overline{x}_0\ a\xi$) irrespective to υm size minimum always by the same value ω_x. This value ω_x is thus optimal radian frequency ω_{xopt}, because it matches the least joint stress.

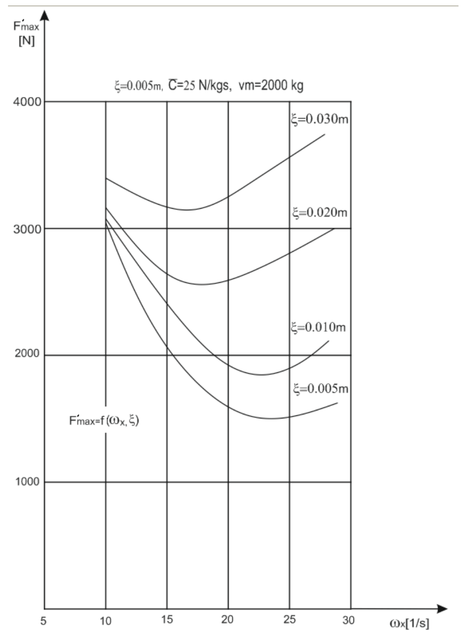
Thus i tis possible to find optimal radian frequency ω_{xopt} for every towing automobile characterized by values $\overline{C}, m, \overline{x}_0$ while selected allowance size ζ . We can see ω_{xopt} = 21,6 s^{-1} on graph 11 and ω_{xopt} = 18,25 s^{-1} on graph 12.

Optimal radian frequency importance consist in the fact, that It is possible to find optimal spring firmness in joint c_{opt} for certain trailer allocated to towing automobile thanks to $\omega_{xopt.}$ It results from finding that in graphs 9, 10, 11 and 12 ω_x changes following spring firmness in joint change c and also from relation (5) according to which it is possible to write

$$c_{opt} = \nu m \omega_{xopt}^2 \cdot \tag{8}$$



Graph 9 Relation $F'_{\max} = f(\omega_x \cdot \xi)$ for $\bar{x}_0 = 0{,}005\ m$ $\bar{C} = 5N.kg^{-1}.s^{-1}$, $vm = 2000\ kg$



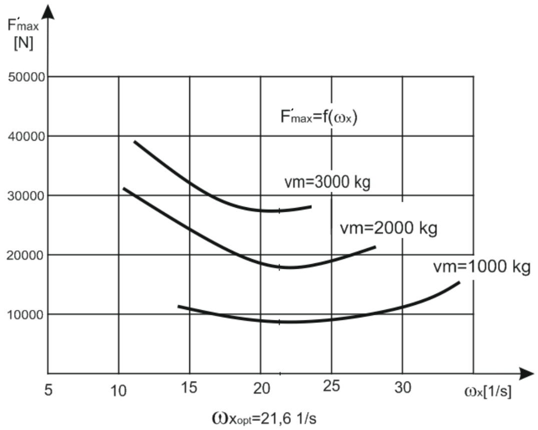
Graph 10 Relation $F'_{\max} = f(\omega_x \cdot \xi)$ for $\bar{x}_0 = 0{,}005\ m$ $\bar{C} = 25N.kg^{-1}.s^{-1}$, $vm = 2000\ kg$

However, trailer´s weight m' is not the only value. In operation its size changes from $m'_{\min}$ (i.e. empty trailer´s weight) to $m'_{\max}$ (i.e. loaded trailer´s weight). Thereby υ and real radian frequency ω_x change and consequently other two values change: the biggest strength in joint and the biggest trailer´s acceleration.

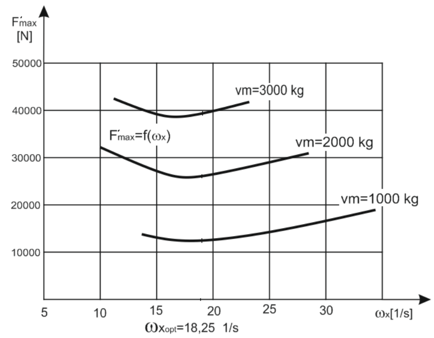
If trailer´s acceleration does not matter, it is favorable to use for optimal firmness c_{opt} setting according the formula (8) and to take into an account the biggest trailer´s weight $m'_{\max}$. Under smaller weights (in the mentioned range $m'_{\min} - m'_{\max}$) the power in joint will be always lower. In some cases and especially in case when there is some special sensitive equipment installed on trailer, trailer´s acceleration strength can be on the contrary critical. It is necessary to take into an account the smallest trailer´s weight while setting optimal firmness c_{opt} so that this acceleration would not reach too big values. There were used only two extreme cases. Having specific project it is necessary to come out from vehicle combination needs, trailer and equipment installed on it, and out of their analysis we can set up conditions for spring firmness in joint.

Having case of towing device project we would proceed by mentioned way. Still, in reality we use appropriate towing automobile that is already equipped with the lifting equipment / including spring mounting/ for projected trailer. In such case we have to check how differentiate real spring firmness in lifting equipment of towing device from calculated optimal firmness. If the required optimal firmness is lower than the real one, it is good to spring back the towing rod / i.e. to place spring between springle of towing rod and own towing rod/. The overall firmness of joint will decline by this serial by-spring sequencing.

If, on the contrary the spring firmness in the towing device is smaller as it is required, it is essential to replace this spring by the one that has bigger firmness or to add another spring to the original one in parallel. Possibilities of such adjustments should be kept in minds already while constructing lifting devices on towing automobiles. Necessity of realizing those adjustments together with the economical point of view should be always evaluated first.



Graph 11 Relation $F'_{\max} = f(\omega_x, \nu m)$ for $\xi = 0{,}01m$



Graph 12 Relation $F'_{\max} = f(\omega_x, \nu m)$ for $\xi = 0{,}02m$

We should deal with such spring firmness in joint and its stating, however the joint stressing should as small as possible. This spring firmness depends on towing vehicle weight transformed into system of units SI as follow c = 100 m ± 300 000 (9)

where weight m comes in kg and spring firmness c comes in N / m.

Formula (9) comes from analysis of the smallest joint loadings having three vehicle combinations. Their weight figures and towing vehicle type are presented in table numb.1. Resulting from the last column of this table all three examined vehicle combinations have almost the same nominal weight of trailer (υ ~ 0,4). Formula (9) does not take into an account size of υ. Lets compare then firmness stated according the formula (9) with the values calculated according the formula (8) by various nominal trailers´ weights.

Table 1 Loading of vehicle combinations´ joint and their impact on weight figures.

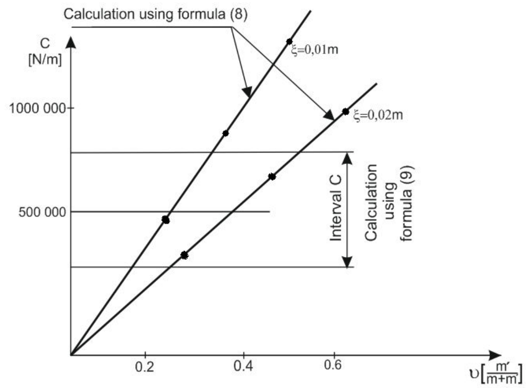
Towing vehicle	**m [kg]**	m' **[kg]**	**Y** $\left[\frac{m'}{m+m'}\right]$
GAZ – 69	5 430	3 500	0,392
ZIL – 133	10 300	6 400	0,383
JAZ – 214	20 000	14 000	0,412

For this comparison, we use cases reported on the graphs 11 a 12. Towing vehicle´s weight is = 5 000 kg and three various nominal trailers´ weights are υ = 0,2,

υ = 0,4, υ = 0,6. It implies from formula (9), that the optimal spring firmness in joint is = 200 000 N/m having medium value c = 500 000 N/m, no matter what the size υ and ζ are. If for stating optimal spring firmness we use the formula (8), it is necessary to take into an account optimal radian frequency of its own oscillations of vehicle combination, stated in graphs 11 and 12. Graph 11 shows that having ζ = 0,01m, $\omega_{xopt} = 21{,}6\ s^{-1}$ and picture 12 having ζ = 0,02 m is $\omega_{xopt} = 18{,}25\ s^{-1}$. Values of optimal spring firmness calculated from the formula (8) are reported in table 2 for both cases. These results are shown on graph 13 c = f (υ) together with firmness range marking, that was calculated out of formula (9). Picture shows that formula (9) does not cover all firmness values by its range calculated from graph (8) for changing value υ. Consequently it is necessary to mention that range of formula (9) is so large, it is possible to make easily serious mistake while its use.

Table 2 Optimal Spring Firmness Values

$\upsilon \left[\dfrac{m'}{m+m'}\right]$	c [N / m]	
	ζ = 0,01 m	**ζ = 0,01 m**
0,2	466 000	333 000
0,4	932 000	666 000
0,6	1 398 000	999 000



Graph 13 Calculation Comparison of Optimal Spring Firmness following the graphs (8) and (9)

$(\upsilon \left[\dfrac{m'}{m+m'}\right])$

Besides these two formula (9) disadvantages in comparison with formula (8) it is necessary to report other two disadvantages. It is particularly the fact, that allowance influence ζ is partially taken into an account only by mentioned extensive range c. Fourth disadvantage is influence neglect of value $\overline{C}$. Method of optimal firmness stating that uses formula (8) takes into an account the influence of υ, ζ, $\overline{C}$, $\overline{x}_0$ values.

Conclusion

From introduced article we can thus draw following conclusions. Formulas (1) and (3) enable simple formulation of the biggest trailer´s acceleration $a'_{\max}$ and the biggest strength in the joint of vehicle combination $F'_{\max}$ mainly because there exist the biggest spring deformation in joint $\overline{x}_{\max}$, which represents the function of several other values and that can be obtained from graphs on pict. 1 – 8. These formulas are then more simple than formula (9), where exists 8 values while expressing the biggest strength. There has been relation of the biggest joint deformation found $\overline{x}_{\max}$, on the radian frequency of its own longitudinal oscillations ω_x. It is easily possible to find the value $\overline{x}_{\max}$,, through the graph system $\overline{\mathrm{x}}_{\max} = f(\omega_x)$ This value is essential for formula (1) and (3) and their calculations. Relatively small influence of spring prestress in joint on the size of the biggest deformation $\overline{x}_{\max}$ [3], [4] has been confirmed. Solution results show applicability of relative trailer´s weight range υ = 0,25 – 0,5. There have been shown various ways of optimal value stating ω_{xopt} (for minimal joint stressing) and by this through formula (8) the way of optimal spring firmness in joint c_{opt} has been shown as well. Essential is to recommend, to adjust firmness in joint while creating the vehicle combination (e.i. while selection of towing automobile for trailer). Firmness in joint should be adjusted so that it would be close to optimal value as much as possible. By comparison of various firmness in joint stated by formulas (8) and (9), there arisen advantages of stating firmness in joint mentioned in this article.

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