On The Infeasibility of Superluminal Velocities as an Extension of Special Relativity

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Abstract

In a recent paper by James Hill and Barry Cox, two new transformations have been proposed between inertial frames that are moving at superluminal speed with respect to each other. It has been shown by Andreka et al. that these transformations do not maintain the constancy of the speed of light in directions other than the line of relative motion. We show that this can also be readily deduced from the invariant equation associated with the Hill-Cox transformations. We further show that, even in one spatial dimension, when we consider two specific speeds v *(subluminal) and* $V = (c^2/v)$ *(superluminal), and consider the inertial frames associated with these two speeds, the space and time coordinates of one inertial frame become the time and space coordinates of another frame respectively.*

Key Words: Special relativity, Lorentz transformation, superluminal velocity, speed of light

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1. Introduction

In a recent paper [1] two transformations as depicted below have been proposed to accommodate superluminal velocities within the frame work of Special Relativity. A rectangular Cartesian frame $K(X,Y,Z)$ and another frame $k(x,y,z)$ moving with constant velocity V relative to the first frame along the aligned X and x directions are considered and the following two possible transformations have been proposed in [1] for the case of superluminal velocity V :

$$x=\frac{X-VT}{\sqrt{\frac{V^2}{c^2}-1}} \quad ; \quad t=\frac{T-\frac{VX}{c^2}}{\sqrt{\frac{V^2}{c^2}-1}} \quad \text{----------------------- (1)}$$

and alternatively

$$x=\frac{-X+VT}{\sqrt{\frac{V^2}{c^2}-1}} \quad ; \quad t=\frac{-T+\frac{VX}{c^2}}{\sqrt{\frac{V^2}{c^2}-1}} \quad \text{--------------------- (2)}$$

The above two transformations are equations 3.16 and 3.18 of the referenced paper [1].

2. The infeasibility of scaling up Hill-Cox formulation to more than one spatial dimension

It has been shown in [2] that the constancy of the speed of light in directions other than the line of motion is not maintained by the Hill and Cox formulations. We offer an alternative explanation for the same below.

The Hill and Cox formulations are associated with the following invariant equation (Eqn. 3) in uni-spatial dimension [1].

$$X^2 - c^2T^2 = -(x^2 - c^2t^2) \text{ ------------------ (3)}$$

When we add another spatial dimension perpendicular to the line of motion and require that the speed of light is the constant c in all directions, the Euclidian distance between two points will replace the uni-dimensional distance X and x in the above equation. Hence the resultant equation in two-dimensional space will take the form

$(X^2 + Y^2) - c^2 T^2 = -((x^2 + y^2) - c^2 t^2)$ ----------- (4)

Equation (4) maintains the speed of light as c in all directions and reduces to equation (3) in the uni-dimensional space formed by the line of relative motion.

Comparing equations (3) and (4), we get

$Y^2 = -y^2$; or Y= y = 0 --------- (5)

Thus no non-zero value is possible for any other spatial dimension, other than the line of relative motion. Therefore, the Hill-Cox formulations are not scalable to more than one spatial dimension which is shown by a different methodology in [2].

A more detailed and rigorous derivation is given in Appendix 1.

3. Transformation between k and an inertial frame k' moving at subluminal speed $v = c^2/V$ with respect to the inertial frame K (in uni-dimensional space)

Let us choose an inertial frame k' that is moving at $v = \frac{c^2}{V}$ with respect to the inertial frame K. To summarize, we have

a) Inertial frame K with associated coordinates (X, Y, Z).
b) Inertial frame k, with associated coordinates (x, y, z) moving at superluminal speed V along the aligned x–X axis with respect to K.
c) Inertial frame k' with associated coordinates (x', y', z'), moving at the specifically chosen subluminal speed $\frac{c^2}{V} = v$ along the aligned x'–X axis with respect to K.

Now the transformation of event coordinates from k' to K is given by the Lorentz (inverse) transformation, as below.

$$X = \frac{x' + v t'}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{----------------------- (6)}$$

$$T = \frac{t' + \frac{v x'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{----------------------- (7)}$$

The transformation of event coordinates from K to k is given by (we reproduce Equation 3.16 from the Hill and Cox paper [1])

$$x = \frac{X - V T}{\sqrt{\frac{V^2}{c^2} - 1}} \quad \text{----------------------- (8)}$$

$$t = \frac{T - \frac{V X}{c^2}}{\sqrt{\frac{V^2}{c^2} - 1}} \quad \text{--------------------- (9)}$$

Substituting for X and T from (6) and (7) into (8) and (9), we get the transformation equations for k' to k. Doing this we get

$$x = \frac{\dfrac{x' + v t'}{\sqrt{1 - \dfrac{v^2}{c^2}}} - V \dfrac{t' + \dfrac{v x'}{c^2}}{\sqrt{1 - \dfrac{v^2}{c^2}}}}{\sqrt{\dfrac{V^2}{c^2} - 1}}$$

$$= \frac{x' + v t' - V\, t' - \dfrac{V v}{c^2}\, x'}{\sqrt{1 - \dfrac{v^2}{c^2}}\ \sqrt{\dfrac{V^2}{c^2} - 1}}$$

Noting that $V\, v = c^2$

We get $x = \dfrac{t'\,(v - V)}{\sqrt{\left(1 - \dfrac{v^2}{c^2}\right)\left(\dfrac{V^2}{c^2} - 1\right)}}$

$$= \frac{t'\,(v - V)}{\sqrt{\dfrac{V^2}{c^2} - 1 - \dfrac{v^2 V^2}{c^4} + \dfrac{v^2}{c^2}}}$$

$$= \frac{t'\,(v - V)}{\sqrt{\dfrac{V^2}{c^2} - 1 - 1 + \dfrac{v^2}{c^2}}}$$

$$= \frac{t'\,(v - V)}{\sqrt{\left(\dfrac{V}{c} - \dfrac{v}{c}\right)^2}} = \pm c\, t'$$

By following a similar procedure we get $t = \pm \dfrac{x'}{c}$

Thus the transformation between k' $\rightarrow$ k interchanges the spatial and time coordinates with appropriate scaling factors.

Thus we have

$$x = \pm \quad c\, t' \qquad \text{------------------------------ (10)}$$

and $$t = \pm \quad \frac{x'}{c} \qquad \text{------------------------------ (11)}$$

By utilizing equation 3.18 of [1], we obtain the same results.

The above analysis may be summarized as below with the specific combination of the velocities V (superluminal) and v (subluminal) such that $V\, v = c^2$.

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} \dfrac{1}{\sqrt{\dfrac{V^2}{c^2}-1}} & \dfrac{-V}{\sqrt{\dfrac{V^2}{c^2}-1}} \\ \dfrac{-\dfrac{V}{c^2}}{\sqrt{\dfrac{V^2}{c^2}-1}} & \dfrac{1}{\sqrt{\dfrac{V^2}{c^2}-1}} \end{pmatrix} \begin{pmatrix} \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} & \dfrac{v}{\sqrt{1-\dfrac{v^2}{c^2}}} \\ \dfrac{\dfrac{v}{c^2}}{\sqrt{1-\dfrac{v^2}{c^2}}} & \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

$\downarrow$ $\downarrow$

Hill and Cox 3.16 K $\rightarrow$ k (SuperLuminal) Inverse of Lorentz k' $\rightarrow$ K (Subluminal)

After doing the matrix multiplication and associated algebraic simplifications, we get

$$\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} 0 & \pm c \\ \pm\dfrac{1}{c} & 0 \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix}$$

$$x = \pm\ c\, t' \quad ; \quad t = \pm\ \frac{x'}{c}$$

In the usual graphical representation, the above concepts may be represented as shown below.

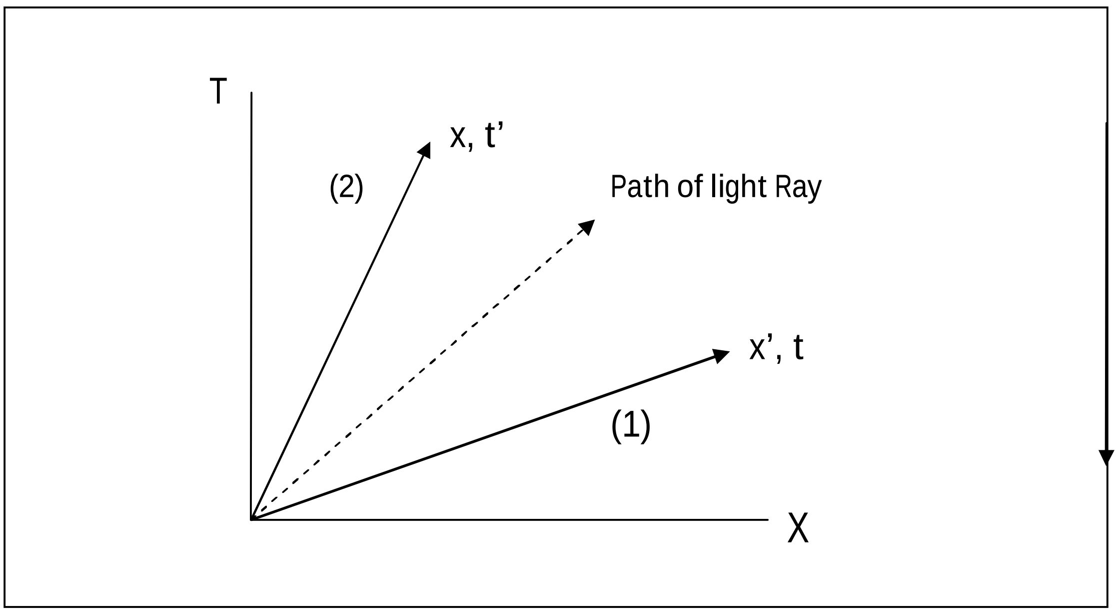

Figure - 1

In Figure 1, X and T represent the space and time axis of the inertial frame K . Line (1) serves as both the spatial axis (x') of the subluminal (with respect to K) frame k' and the time axis (t) of the superluminal (with respect to K) frame k. Similarly Line (2) serves as both the spatial axis (x) of the superluminal frame k and the time axis (t') of the subluminal frame k'. Thus between k and k' we find that the space and time axes have interchanged.

A graphical representation of the event coordinate relationship between inertial frames k and k' is depicted in Figure 2.

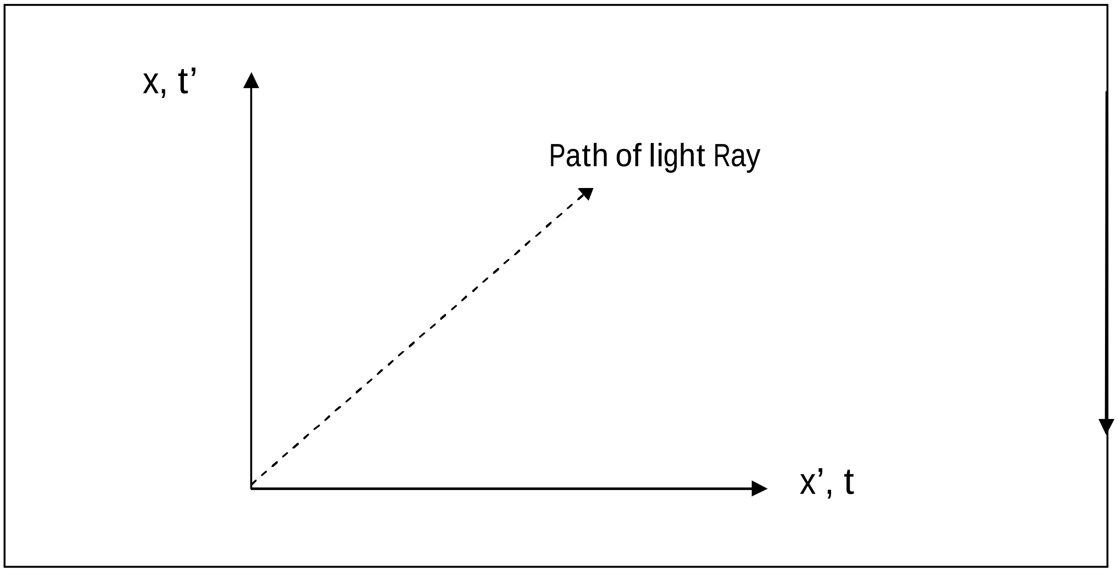

Figure - 2

4. Conclusion

With the proposed Hill and Cox [1] formulation for superluminal velocities in uni-dimensional space, we find that two inertial frames travelling at v and V with respect to a third inertial frame such that $v V = c^2$, have their space and time axis interchanged. The time coordinate of one frame becomes the space coordinate of the other frame and vice versa. That is $x' = c t$; $t' = (x/c)$; Thus we would find that if an observer A in k moves back and forth on the x axis, observers in k' would interpret this as A is moving from past to present and present to past freely at will. Further, the Hill and Cox transformations do not maintain the constancy of the speed of light in directions other than the line of relative motion [2]. This is easily deducible from the invariant equation (3) associated with the Hill-Cox formulations, as the scaling up to the next spatial dimension is not feasible as shown in Section 2.

References

[1] Hill J.M., Cox B.J. (2012) Proc. R. Soc. A October 2012 "Einstein's Special Relativity Beyond the speed of light" Published Online 3rd October, 2012 doi: 10.1098/rspa.2012.0340

[2] Andreka et al. (2013) A note on "Einstein's Special Relativity Beyond the speed of light by James M. Hill and Barry J. Cox" http://arxiv.org/pdf/1211.2246v2 17, Feb, 2013

Appendix 1

In a plane (of 2 spatial dimensions) a light ray originating at x = 0,y = 0,t =0 (0,0,0) and propagating to a space-time point (x, y, t) obeys the equation
$(x^2+y^2)^{1/2} = ct$ or $(x^2+y^2)^{1/2} - ct = 0$ ------------------- (A1)

If a light ray originates at (x, y, t) and reaches (0,0,0) then it obeys the equation
$(x^2+y^2)^{1/2} = -ct$ or $(x^2+y^2)^{1/2} + ct = 0$ ------------------- (A2)

Thus if a light ray creates two events (0,0,0) and (x, y, t), in the case t > 0 it obeys equation A1 and in the case t < 0 it obeys equation A2 with $(x^2+y^2)^{1/2} > 0$.

Thus when we have a light ray creating two events (0,0,0) and (x, y, t), (the propagation may be in either direction) we have
Either $(x^2+y^2)^{1/2} - ct = 0$ or $(x^2+y^2)^{1/2} + ct = 0$,
which implies $[(x^2+y^2)^{1/2} - ct]\,[(x^2+y^2)^{1/2} + ct] = 0$

In other words
$x^2 + y^2 - c^2t^2 = 0$ ----------------------- (A3)

For another plane in relative motion with respect to this plane and with coordinates denoted by X, Y, T we have for the same light ray, by the same logic
$X^2 + Y^2 - c^2T^2 = 0$ ------------------------ (A4)

So whenever equation A3 is true, equation A4 is also true.
This is possible only when
$(x^2+y^2 - c^2t^2) = \lambda\,(X^2+Y^2 - c^2T^2)$ ----------------- (A5)

So any transformation equation that transforms (x,y,t) to (X,Y,T) shall obey equation A5 in order that the propagation of light happens at speed *c* in both the planes that are in relative motion.

Computing possible Values of λ

Substituting x'= – x and X'= – X, the observations of inertial frames K and K' interchange. That is if K' is moving at v along x axis with respect to K, then after making the substitutions x' = – x and X' = – X, K now is moving at v with respect to K'. Therefore by the principle of equivalence, we have

$\lambda(x'^2+y^2 - c^2t^2) = (X'^2+Y^2 - c^2T^2)$ ----------------- (A5a)
Since $x'^2 = x^2$ and $X'^2 = X^2$ the above equation may be re-written as
$\lambda(x^2+y^2 - c^2t^2) = (X^2+Y^2 - c^2T^2)$ ----------------- (A5b)
This may be re-written as
$(x^2+y^2 - c^2t^2) = (1/\lambda)\,(X^2+Y^2 - c^2T^2)$ ----------------- (A5c)
Comparing equations A5 and A5c, we get $\lambda = (1/\lambda)$ or $\lambda^2 = 1$
or $\lambda = \pm 1$ ------------------ (A5d)
On the line of relative motion y = Y = 0 and we have
$(x^2 - c^2t^2) = \lambda\,(X^2 - c^2T^2)$ ---------------- (A6)
The Hill and Cox formulations [1] specify that $\lambda = -1$ or
$(x^2 - c^2t^2) = -(X^2 - c^2T^2)$ ------------------- (A7)
[Note: The Lorentz Transformations uses λ=1]

So for the Hill and Cox formulations λ is –1 for equation A6 and therefore has to be the same for equation A5 as well (because the introduction of a small non-zero value for y cannot switch λ from –1 to +1; this will imply a sudden discontinuity).
Therefore equation A5 becomes (for the Hill and Cox formulations)
$(x^2+y^2 - c^2t^2) = -(X^2+Y^2 - c^2T^2)$ -------------------- (A8)
Comparing equations A7 and A8, we get
$y^2 = -Y^2$ ------------------ (A9)
or y = Y = 0 -------------- (A10)

Thus the Hill and Cox formulations cannot have a non-zero spatial dimension perpendicular to the line of relative motion when the speed of light propagation is the constant *c* in both the inertial reference frames. In [2] it is shown that when y and Y are non-zero, the Hill and Cox formulations fail to maintain the constancy of light speed. This is consistent with the above derivation.