Application of Queuing Theory to Vehicular Traffic at Signalized Intersection in Kumasi-Ashanti Region, Ghana

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Abstract

Traffic congestion is a growing problem in many metropolitan areas as it increases travel time, air pollution, carbon dioxide (CO_2) emissions and fuel use because cars cannot run efficiently. This paper seeks to model the vehicular traffic flow and explore how vehicular traffic could be minimized using queuing theory in order to reduce the delays on roads in the Kumasi metropolis of Ghana. The Oforikrom traffic intersection in the Kumasi metropolis of Ghana is currently operating with one service channel each from the various routes to the intersection. The results showed that traffic intensity, $\rho<1$ for all sessions, a condition that suggests a perfect traffic system. Consequently, smooth flow of traffic was shown since the server at each channel was able to serve more than the cars in waiting queue when servers resume work. Again, it was found that heavy traffic occurs in the evening. Stakeholders can task Motor Traffic Transport Unit (MTTU) to check that drivers desist from such practices so that there will be free flow of traffic in the evening and also promote the use of bikes, which apart from serving as a form of exercise also helps to reduce fuel consumption thereby saving money for the Government to tackle problem of other sectors of the economy. Finally, the government of Ghana could introduce a public transport system so that people do not travel with private cars to their places of work to reduce congestion on the roads, which in turn boosts productivity.

Keywords: Queuing Theory, Traffic Congestion, Traffic Light System and Transportation

1. Introduction

Queuing theory is the mathematical study of waiting lines, or the act of joining a line (queues). In queueing theory a model is constructed so that queue lengths and waiting times can be predicted (Sundarapandian, 2009). The issue of queuing has been a subject of scientific debate for there is no known society that is not confronted with the problem of queuing. Wherever there is competition for limited resource queuing is likely to occur. The role of transportation in human life cannot be overemphasized. According to Intikhab et al. (2008), efficient transportation system plays an important role in catering for the daily necessities in the lives of the citizens. These include access to amenities and services that are central to the lives of all individuals, like employment, education, health services and leisure. At the individual level, Wane (2001, p.1) also points out that ‘transportation is a crucial factor for urban insertion since it gives access to economic activity, facilitates family life and helps in spinning social networks.

Consequently, in an attempt to acquire vehicles to enable individuals traveling faster and carry out daily activities easily, cities in the world now witness tremendous motorization during the recent times, especially since 1988 global car population have exceeded 400 million (Walsh, 1990). The reason for this phenomenon, according to Dimitriou (1991) is that in both the Transportation "currently accounts for 23 percent of the world's greenhouse-gas emissions," and according to Brad Plumer who writes at the Washington Post “and most of that unconventional oil is significantly dirtier, from a CO_2 perspective, than the traditional stuff. If people in the developing world keep buying vehicles then simple upgrades in fuel-efficiency alone are not going to be enough to stop a steady uptick in global temperatures"(Tencer, 2011).

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Traffic congestion is a phenomenon of increased disruption of traffic movement on an element of the transport system. It is most visible when the level of demand for movement approaches or exceeds the present capacity of the element (Taylor, 1999). As Taylor et al. (2000) argued, traffic congestion presents a common, if not inevitable, facet of traffic activity in a region, particularly in urban areas. It is also believed that the high volume of vehicles, the inadequate infrastructure and the irrational distribution of the development are the main reasons for increasing traffic jam (O'Toole, 2012). The Government in its attempt to salvage the situation has been expanding road networks in almost all our major cities but the more they expand, the more people import cars for their domestic use. (Ghanaian Chronicle, 2007).

Quddus et al. (2010) argued that the level of traffic congestion does not affect the severity of road crashes on the M25 motorway. The impact of traffic flow on the severity of crashes, however, showed an interesting result. Yannis et al. (2010) developed ordered logit models to provide insight into the human-factors' aspect of the introduction of advanced technologies with respect to these more sensitive segments of the driver population. Other studies, Chao et al. (2009), noted that traffic congestion has little or no impact on the frequency of road accidents on the M25 motorway. Glen et al. (2003) showed how congestion-reduction strategies can induce additional traffic as a result of economic benefits. Qingyu et al. (2007) suggested a base for the implementation of urban road congestion pricing and other travel demand management strategies after analyzing the production mechanism of urban traffic congestion: travelers overlook the negative externality of urban traffic congestion and then join the congested queues.

Meanwhile, Simulation based analysis on simple network topologies showed that the local de-congestion protocol can enhance road capacity and prevent congestion collapse in localized settings (Vipin et al., 2012). Jayaram & Lincoln (2007) identified two polar cases. In one case, the estimation of this number is trivial and therefore enables further analysis, including the quantification of the relative impact of congestion on distribution costs and the deduction of empirically testable hypotheses. The other case is considerably less tractable; we consider a specific instance of this case that was broadly relevant to one of the companies that participated in the research. Further, Strasbourg et al. (2008) adopted laboratory experiments designed to study the impact of public information about past departure rates on congestion levels and travel costs. The design was based on a discrete version of Arnott et al. (1990) bottleneck model. In all treatments, congestion occurs and the observed travel costs were quite similar to the predicted ones. Subjects' capacity to coordinate was not affected by the availability of public information on past departure rates, by the number of drivers or by the relative cost of delay. This seemingly absence of treatment effects was confirmed by finding that a parameter-free reinforcement learning model best characterizes individual behavior. Luke & Richard (2006) obtain detailed data on road traffic crash (RTC) casualties, by severity, for each of the eight state and territory jurisdictions in Australia and used these to estimate and compare the economic impact of RTCs across these regions. The paper makes two fundamental contributions: (i) it provided a detailed breakdown of estimated RTC casualties, by state and territory regions in Australia, and (ii) it presented the first sub-national breakdown of RTC costs for Australia.

Kumasi is faced with problem of vehicular traffic congestion which is becoming more serious day after day. Ten years ago it took about fifteen (15) minutes to travel from Oduom to central market but today it takes about one hour by car. Accordingly, this article seeks to build a basic model of vehicular traffic based on queuing theory. Therefore, this will determine the best times for the red, amber and green lights to be either on or off in order to reduce traffic congestion at the Oforikrom intersection in Kumasi, the Ashanti Region of Ghana.

2. Materials and Method

This section introduces the data sources, discusses the $M/M/1/\infty$ queuing model which this article uses to model the vehicular traffic flow and to explore how vehicular traffic could be minimized using queuing theory in order to reduce the delays on roads in Kumasi, capital of Ashanti Region, Ghana

2.1 Data Sources

This paper uses data recorded within the working periods of morning, afternoon and evening where roads are normally congested at the Oforikrom intersection in Kumasi, Ghana.

The data of traffic from Asokwa to Oforikom was not considered because only negligible size of the traffic used the route due to road construction being carried at the time of the research.

Table 1: Traffic data at Oforikrom Intersection

LOCATION	SESSION	ARRIVAL		SERVICE	
		Average No. Cars	Time(min)	Average No. Cars	Time(min)
Tech-to-Oforikrom	MORNING	45	1:34	60	1:30
Amakom-to-Oforikrom	MORNING	40	1:34	50	1:30
Aboabo-to-Oforikrom	MORNING	11	2:30	12	0:34
Tech-to-Oforikrom	AFTERNOON	30	1:34	50	1:30
Amakom-to-Oforikrom	AFTERNOON	36	1:34	50	1:30
Aboabo-to-Oforikrom	AFTERNOON	11	2:30	10	0:34
Tech-to-Oforikrom	EVENING	40	1:34	55	1:30
Amakom-to-Oforikrom	EVENING	45	1:34	60	1:30
Aboabo-to-Oforikrom	EVENING	13	2:30	15	0:34

2.2 M/M/1/∞ Queueing System

This article will focus on the M/M/1 queuing system. M/M/1 refers to negative exponential arrivals and service times with a single server. This is the most widely used queuing systems in analysis as pretty much everything is known about it. M/M/1 is a good approximation for a large number of queuing systems. Suitability of M/M/1 queuing is easy to identify from the server standpoint. For instance a single transmit queue feeding a single link qualifies as a single server and can be modeled as an M/M/1 queuing system.

2.3 Poisson Arrivals and Process

M/M/1 queuing systems assume a Poisson arrival process. This assumption is a very good approximation for arrival process in real systems that meet the following rules:

1. The number of customers in the system is very large.
2. The impact of a single customer for the performance of the system is very small, that is, a single customer consumes a very small percentage of the system resources.
3. All customers are independent, i.e. their decisions to use the system are independent of other users.

Figure 1: M/M/1 Node

2.4 Cars on a Highway

These assumptions are fairly general, so they apply to a large variety of systems. For instance cars entering a highway could follow these assumptions as follows: Total number of cars driving on the highway is very large.

1. Total number of cars driving on the highway is very large.
2. A single car uses a very small percentage of the highway resources.
3. The decision to enter the highway is independently made by each car driver.

The above observations mean that assuming a Poisson arrival process will be a good approximation of the car arrivals on the highway. If any one of the three conditions is not met, one cannot assume Poisson arrivals (eventhelix.com, 2013).

2.5 Probability Density Distribution

This probability density distribution equation for a Poisson process describes the probability of seeing **n** arrivals in a period from 0 to **t**.

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t} \qquad (1)$$

Where:

- **t** is used to define the interval 0 to t
- **n** is the total number of arrivals in the interval 0 to t.
- **λ** is the total average arrival rate in arrivals/sec.

2.6 Negative Exponential Arrivals

This equation gives information about how the probability is distributed over a time interval. The probability of no arrivals taking place over a given interval is obtained by substituting n with 0, and arrives at getting the following equation:

$$P_0(t) = e^{-\lambda t} \qquad (2)$$

Equation two (2) shows that probability that no arrival takes place during an interval from 0 to t is negative exponentially related to the length of the interval.

2.7 Poisson Service Times

In an M/M/1 queuing system we assume that service times for customers are also negative exponentially distributed (i.e. generated by a Poisson process). Unfortunately, this assumption is not as general as the arrival time distribution. A queuing discipline determines the manner in which the exchange handles calls from customers. The most common queue discipline is "first- come, first served", abbreviated as FCFS, in some inventory applications, the same rule is called "first in- first out" and abbreviated as FIFO.

This principle also serves customers one at a time, however the customer with the shortest waiting time will be served first.

2.8 Mean Performance Parameters Of The Queue M/M/1/∞

The equations describing a M/M/1 queuing system are fairly straight forward and easy to use. First we define the traffic intensity, ρ (sometimes called occupancy). It is defined as the average arrival rate (λ) divided by the average service rate (μ). For a stable system the average service rate should always be higher than the average arrival rate. (Otherwise the queues would rapidly race towards infinity). Thus ρ should always be less than one. Also note that we are talking about average rates here, the instantaneous arrival rate may exceed the service rate. Over a longer time period, the service rate should always exceed an arrival rate.

$$\rho = \frac{\lambda}{\mu} \qquad (3)$$

Mean number of customers in the system (N) :can be found using the following equation

$$N = \sum_{i=0}^{\infty} i\rho_i = \frac{\rho(1-\rho)}{(1-\rho)^2} = \frac{\rho}{1-\rho} or \;\; N = \frac{\lambda}{\mu - \lambda} \qquad (4)$$

Mean number of customers in queue (prior to service)

$$N_q = \sum_{i=0}^{\infty} (i-1)\rho_i = \frac{\rho}{(1-\rho)} - (1-(1-\rho)) = \frac{\rho^2}{(1-\rho)} \qquad (5)$$

From equation five (5) above, as ρ approaches 1 the number of customers would become very large. This can be easily justified intuitively, ρ will approach 1 when the average arrival rate starts approaching the average service rate. In this situation, the server would always be busy hence leading to a queue build up (large N).

Lastly we obtain the total (mean) waiting time (including the service time):

$$T = W = \frac{N}{\lambda} = \frac{\rho}{(1-\rho)\lambda} = \frac{1}{\mu(1-\rho)} = \frac{1}{\mu-\lambda} \quad \text{or} \quad T = W_q + \frac{1}{\mu} \tag{6}$$

Mean time spent waiting in queue (prior to service)

$$T_q = W_q = \frac{\rho}{\mu(1-\rho)} \tag{7}$$

Again we see that as mean arrival rate approaches mean service rate, the waiting time becomes very large (eventhelix.com, 2013).

3. Results and Discussions

The general objective of this study is to build a basic model of vehicular traffic based on queuing theory and then use it to determine the best times for the red amber and green lights to be either on or off in order to reduce traffic congestion at the Oforikrom intersection in Kumasi, in the Ashanti Region Ghana.

3.1 Computation of Queuing Parameters

Computations of parameters to determine the behaviour of traffic were done using the M/M/1 queuing model. It is assumed that time interval between successive arrivals and serving time is independent and identically distributed. In any sufficient interval of time at most only one arrival can occur. The system is also assumed to reach a steady state, a condition that the rate of arrival and service is constant. The queuing discipline observed was first-come-first-served (FCFS).

The data from Table 1 were used to obtain the mean performance parameters of the M/M/1 queue model as follows:

Morning Session (Tech-to-Oforikrom)

Arrival Rate $\lambda = \frac{45}{1.34} = 34, cars/\min$

Service Rate $\mu = \frac{60}{1.30} = 47, cars/\min$

Traffic Intensity $\rho = \frac{34}{47} = 0.7234$

Mean time spent in the system $w = \frac{1}{47(1-0.7234)} = 0.0769, \min$

Mean time spent waiting in queue $w_q = \frac{0.7234}{47(1-0.7234)} = 0.0556, \min$

Mean number of Cars in the system $N = \frac{0.7234}{(1-0.7234)} = 2.610 = 3$

Mean number waiting in queue $N_q = \frac{(0.7234)^2}{(1-0.7234)} = 1.887 = 2$

The same procedure was used to obtain the values from the various routes as shown in Table 2 below.

Table 2: The Situation of Traffic at Oforikrom Intersection

LOCATION	SESSION	ARRIVAL RATE	SERVICE RATE	TRAFFIC INTENSITY	MEAN NO. OF CARS IN THE SYSTEM	MEAN NO. OF CARS WAITING IN QUEUE	MEAN TIME SPENT IN SYSTEM	MEAN TIME SPENT IN QUEUE
Tech	Morning	34	47	0.7234	3	2	0.0769	0.0556
Amakom	Morning	30	39	0.7692	4	3	0.1111	0.0855
Aboabo	Morning	5	36	0.1389	1	1	0.0323	0.0045
Tech	Afternoon	23	39	0.5897	2	1	0.0625	0.0369
Amakom	Afternoon	27	39	0.6923	3	2	0.0833	0.0577
Aboabo	Afternoon	5	33	0.1515	1	1	0.0357	0.0054
Tech	Evening	30	43	0.6977	3	2	0.0769	0.0537
Amakom	Evening	34	47	0.7234	3	2	0.7234	0.0556
Aboabo	Evening	6	45	0.1333	1	1	0.0256	0.0034

All the routes lead to Oforikrom traffic intersection. The content of Table 2 is the results of the analysis of data recorded in Table 1. They were collected within the working hours from morning, afternoon and evening. The traffic intersection is currently operating with one service channel each from the various routes to Oforikrom intersection. It is clear from the results that $\rho<1$ for all sessions.

4. Conclusion

Critical analysis of the data collected at Oforikrom intersections revealed a smooth flow of traffic and perfect system, since the server at each channel was able to serve more than the cars in waiting queue when servers resume work. Moreover, there seems to be heavy traffic in the evening. It was observed that in the evening Motor traffic transport unit (MTTU) do not work and the drivers of commercial cars take advantage of the situation, park and offload/on-load passengers at unauthorized places very close to the traffic signals, impeding the flow of traffic when servers resumed work. This situation is less or non-existent in the morning/afternoon.

Government can task MTTU to check that drivers desist from such practices so that there will be free flow of traffic in the evening. This is because these practices disrupt the flow and may cause traffic to degenerate into a disorganized mess. We also suggest that the public transport system should be introduced in Ghana so that people do not travel with cars to their places of work. Thus, use of public transport will be reducing congestion on the roads, which in turn boosts productivity. Also, stakeholders can promote the use of bikes, which apart from serving as a form of exercise, helps to reduce fuel consumption thereby saving money for the Government to tackle problem of other sectors of the economy.

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