

## The Prevalence and Effect of Mathematical Ability on Response Aberrance among Secondary School Students in Osun State

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### Abstract

*The study examined the Osun State secondary school students' response aberrance in Mathematics and determined the differences in students' response aberrance in Mathematics among Osun State secondary school students in accordance to mathematical ability. These were with a view of improving the validity, reliability and usability of test scores in the schools. The survey research design was adopted for the study. The study population consisted of all students in Osun state Secondary Schools. A sample of 300 students was selected from ten Local Government Areas (LGAs) in Osun state. From each of the LGAs, three public schools were selected randomly. A total of 20 students were selected purposively from each of the schools according to their mathematical ability. Data were collected using an instrument entitled Mathematics Achievement Test (MAT). The response aberrance values for each students were computed with aberrance indices  $W^*$  (within ability index) and  $B^*$  (beyond ability index) using MATLAB. Data collected were analyzed using descriptive, ANOVA and Scheffe Multiple Comparison. The results of the analysis showed there is response aberrance in Mathematics among secondary school students with mean of 1.8931 and 4.0651 which is more than 0.5 for the two aberrance indices  $W^*$  and  $B^*$  respectively. The results also showed that there was significant difference in the response aberrance as measured by  $W^*$  and  $B^*$  based on students' academic ability ( $F=45.312, p<.05, df = 2$ ) and ( $F= 5.886, p< .05, df= 2$ ). The study concluded that there was response aberrance in Mathematics and that students' mathematical ability has effect on the severity of the response aberrance among secondary school students.*

### Introduction

Aberrance in a set of test responses occurs when the student's response pattern on some questions are inconsistent with demonstrated knowledge for other test questions on the exam. The simplest example of aberrance is when the student is able to answer difficult questions correctly, but is unable to answer easy questions correctly. In addition to testing irregularities, other typical behaviours can contribute to aberrance. These other behaviours include fatigue, poor preparation, illness, running out of time, lack of motivation, guessing, differential test preparation (knowing some content well, but not knowing other content), and so forth. Hence, aberrance must be interpreted carefully. Response aberrance refers to the tendency of an examinee to deviate from a particular response pattern. It is the deviation from an expected response pattern. If an examinee misses many easy items, but correctly responds relatively to many hard items, aberrance or misfit of the resulting response pattern occurs. Statistical aberrance in the test response pattern may indicate a testing irregularity. For example, if the student gets help answering some questions and not others, the student's responses may reveal that the test was taken in more than one mode (i.e., the mode of being assisted as well as the mode of working according to one's ability). Under normal circumstances, a student takes the test in a single mode corresponding to his or her knowledge. The bimodal aberrance statistic used by Caveon (2005), measures when two test-taking modalities are present in the test responses. Aberrance could be "Within Ability Aberrance" that is when an examinee responds wrongly to an easy items or "Beyond Ability Aberrance" that is when an examinee responds correctly to a difficult items. Literatures have pointed out several behaviour patterns that are exhibited by some examinees. Some of this behaviour pattern may not be expected from examinees. For example, a low ability examinee who answers several very difficult items correctly would represent an unexpected occurrence. In broad, general terms, aberrant response patterns can be defined as response patterns that defy some expectation.

Of course, the primary challenge in this endeavor is defining a level of expectation so that a judgment can be rendered regarding the degree of aberrance associated with an examinee's response pattern. A large number of person-fit indicators have been developed for the purposes of identifying aberrant response patterns (Karabatsos, 2003). Many of these techniques compare observed response patterns to expected outcomes defined by a particular model. These values are compared, and person misfit occurs where observed item response patterns are incongruous with what is implied by the model (Meijer, 1996; Meijer, Muijtjens, & van der Vleuten, 1996; Meijer & Sijtsma, 2001). However, depending on the context of measurement (CTT, IRT, or FA), methods for defining expectation in a response pattern and measuring deviations from expectation differ. The impact of ability of the examinees in determining response pattern cannot be overemphasized. Chen (2004) included ability in her study of effects of test anxiety on response aberrance. It was found out that low ability students produce more irregular responses than high ability students in a situation where there is time pressure. According to Chen (2004), "one should not examine response aberrance without taking into account student ability level and test speediness". Therefore in this study, effort was made to look into the relationship between response aberrance and mathematical ability among secondary school students in Osun State. Since students' progress in learning is usually measured with their total scores which may give fake impression about the students, there is the need to look into their response pattern.

Over the years, students have been promoted, selected based on total scores in test but research have revealed that it is possible for a weak students to get high scores in a test and for a good students to get low scores through various means like cheating, plodding, guessing etc. Performance in Mathematics over the years in Osun State has shown that many students who are above average academically perform below expectation in Mathematics test while some students below average perform brilliantly. This indicates that there is aberrance in their response. Some students may pass the Mathematics exam through act of malpractice which may show fake impression about their ability. Karabatsos (2003) in his study commented that high test scores can lead to un-qualified individuals being enrolled into an educational program (e.g., undergraduate, graduate, or professional), or being awarded an educational degree. On the other hand, qualified individuals with spuriously low test scores may be unfairly excluded from academic programs, or unfairly denied a degree. If test items in an exam are ordered in difficulty from the easiest to the hardest, it is generally expected that an examinee's response would exhibit a sequence of a lot of 1's (correct answers) followed by a sequence of a lot of 0's (wrong answers). If an examinee misses many easy items, but responds correctly to many difficult items, then one would assert that deviation from the expected pattern, or aberrance, has occurred.

Moreover, abnormal response style may result to aberrance response pattern. Some examinees may be too slow in responding to the items in a test which may lead to inability to complete the test while some may have problem in starting the test, this may lead to having incorrect responses at the beginning of the test. Also, some examinees may be so careless that they can commit many blunders while responding to a test especially in shading the answer sheet if the items are multiple choice questions. Response pattern especially for secondary school Mathematics contains valuable information not provided by the total score of a testee and this information can be used to identify potentially inaccurate scores. Thus, response aberrance can lead to misuse of information generated through scores. Few empirical studies are directed towards the influence of students' mathematical ability on response aberrance among secondary school students in the study area. Therefore, there is need to ascertain the relationship between the variable.

### ***Objectives of the Study***

The specific objectives of the study are to

- (a) Examine Osun State secondary school students response aberrance in Mathematics;
- (b) Compare the differences in students response aberrance in Mathematics among Osun State secondary school students based on ability.

### ***Research Questions***

1. What is the prevalence of response aberrance in Mathematics among Osun State secondary school students?

### ***Research Hypotheses***

1. There is no significant difference in the response aberrance in Mathematics among secondary school students based on ability

### **Method**

The population of the study comprised senior secondary school students in Osun State. It consisted of students who have spent more than four years in secondary school (Senior Secondary School 2 students). The study sample consisted of 300 students. Ten Local Government Areas (LGAs) were randomly selected from the three senatorial districts in Osun State. Three Local Government Areas were selected randomly from Osun Central, four from Osun East and three from Osun West. A total of three public schools were randomly selected in each of the LGAs. From each of the school, 10 students were selected according to their mathematical ability. The sampled students were classified into low, average, and high ability level based on their performance in Mathematics in the last school exams. Students with B3 and above were classified as having high academic ability, while those with grade that ranged between C6 and C4 were classified as having average academic ability and those with grades below C6 were classified as having low academic ability.

### **Research Instrument**

A self-developed Mathematics Achievement Test (MAT) was used for collecting data for the study. The MAT provided data on examinee response pattern. The MAT was a multiple choice question on Mathematics which was developed by researcher according to current Ministry of Education Mathematics curriculum for senior secondary school with the help of some Mathematics teachers in public schools. It contained 40 items, with equal numbers of easy and difficult questions. Each question provided four multiple choice options. The items of the test were arranged from the easiest to the most difficult questions. There was equal number of easy and difficult items. Any correct response in MAT was scored as "1" while the incorrect response was scored as "0". The pilot study was conducted to show the validity of the instrument. The Pearson correlation between the pre-test and post-test scores of the MAT was 0.68 which is moderate. This correlation value indicates that the MAT is useful for the stated purpose. The result indicates that MAT has high criterion-related validity.

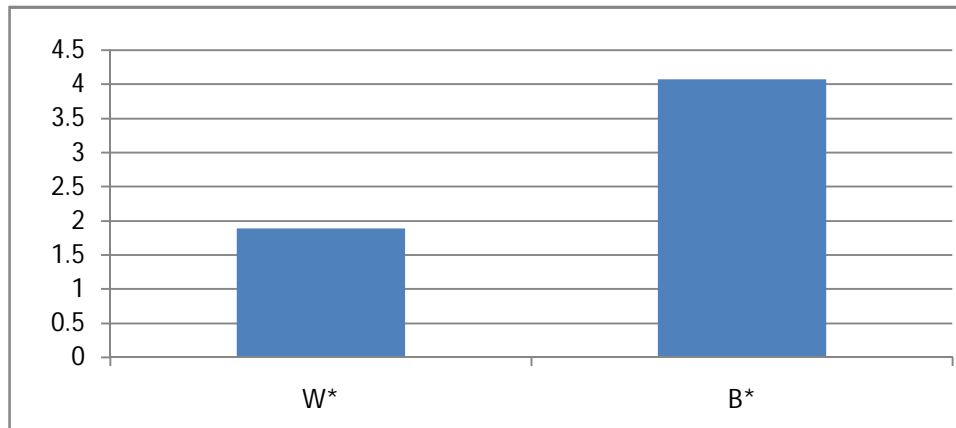
### **Result**

**Research Question 1:** What is the prevalence of response aberrance in Mathematics among Osun State secondary school students?

Before arriving at answer to the research question above, Mathematics Achievement Test (MAT) was administered on the sampled students. The responses were scored and the aberrance index scores were computed for each of the examinee. Two Aberrance indices W\* (within-ability/caution index) and B\* (beyond-ability/surprise index) were computed for each student to measure the response aberrance through MATLAB program. Almost all the students have aberrance value that is more than 0.5 and according to D'costa (1993); the value above 0.5 indicates that there is aberrance in the response pattern. Most of the examinees aberrance values are more than 0.5 which is a signal that there is prevalence of response aberrance in Mathematics among Osun State secondary school students.

#### **The descriptive statistics of the aberrance values**

Aberrance Indices	Mean	SD	Min	Max
(N = 600)				
W*	1.8931	1.6273	0	9.8974
B*	4.0651	3.0650	0	14.4974



**Figure 2: Plot of Response Aberrance in Mathematics**

From table 1 and figure 1 above, W\* has a mean of 1.8931, standard deviation of 1.6273, minimum value of 0, and a maximum value of 9.8974 while B\* has mean value of 4.0651, standard deviation of 3.0650, a minimum value of 0 and a maximum value of 14.4974. The table also shows that the B\* has a higher value of mean and standard deviation ( $\bar{x}=4.0651$ ,  $SD=3.0650$ ) than W\* values ( $\bar{x}=1.8931$ ,  $SD=1.6273$ ) which means that many examinees performed very well beyond their ability level and some also perform below their ability level which indicates that there is aberrance in their response. The minimum value of W\* (Min=0) is the same with the minimum value of B\* (Min=0) while the maximum of B\* (Max=14.4974) is higher than the maximum value of W\* (Max=9.8974).

The results from the Table1 showed that there is prevalence of response aberrance in Mathematics among Osun State secondary school students. The mean value of beyond ability level (B\*) and the within ability level (W\*) aberrance indices are greater than 0.5 which implies that the students deviated in their response pattern. The mean of 4.0651 for B\* and the mean of 1.8931 in the above table is more than 0.5. It shows that many students got most of the difficult questions right while some students missed the easy items which means that the students have deviated from the expected response pattern. Therefore, the result indicates that there is prevalence of response aberrance in Mathematics among Osun State secondary school students.

**Hypothesis 1:** there is no significant difference in the response aberrance as measured by W\* and B\* based on students’ academic ability level.

To test this hypothesis, students’ performance in Mathematics in the last school examination was used to classify the students to low, average and high ability levels. Students with B3 and above were classified as having high academic ability, while those with grade that ranged between C6 and C4 were classified as having average academic ability and those with grades below C6 were classified as having low academic ability. The response aberrance value as measured by W\* and B\* were then sorted into academic ability level and with the aid of one-way ANOVA the differences were determined. The descriptive statistical results were as presented in Table 3.

**Table 3: Descriptive Statistic for Aberrant indices by Ability level**

Ability	N	W*		B*	
		$\bar{x}$	SD	$\bar{x}$	SD
Low	94	1.223	1.235	4.772	3.316
Average	151	0.853	.4008	5.289	2.713
High	55	2.339	1.528	3.701	2.792
<b>Total</b>	300	1.241	1.129	4.840	2.976

Table 3 showed that the mean and SD of within-ability/caution index (W\*) values for students with low, average and high academic ability respectively were ( $\bar{x}=1.223$ ,  $SD= 1.235$ ), ( $\bar{x}=0.853$ ,  $SD= 0.401$ ), and ( $\bar{x}= 2.339$ ,  $SD= 1.528$ ).

Table 3 also showed that the mean and standard deviation of beyond-ability/ surprise index (B\*) values for students with low, average and high academic ability respectively were ( $\bar{x}=4.772$ ,  $SD=3.316$ ), ( $\bar{x}=5.289$ ,  $SD=2.713$ ) and ( $\bar{x}=3.701$ ,  $SD=2.792$ ). The result as presented in Table 3 showed that when the W\* is considered, students with high ability exhibited higher response aberrance. However, with the W\* and B\* mean values that is greater than 0.5, all students irrespective of academic ability exhibited the two examined response aberrance. Thus a one-way ANOVA statistics was used to determine the difference in the mean aberrance as exhibited by the students based on their academic ability level. Table 4 presented the result.

**Table 4: One-Way ANOVA Showing the Difference in Students’ Response Aberrance Based on Academic Ability**

		Sum of Squares	Df	Mean Square	F	Sig.
W*	Between Groups	89.095	2	44.547	45.312	.000
	Within Groups	291.988	597	.983		
	Total	381.083	599			
B*	Between Groups	100.916	2	50.458	5.886	.003
	Within Groups	2528.733	597	8.572		
	Total	2629.649	599			

The results as presented in Table 4 showed that the difference in the response aberrance exhibited by the students as measured by each of W\* ( $F = 45.312$ ,  $df_1 = 2$ ,  $df_2 = 597$ ,  $p < .05$ ) and B\* ( $F = 5.886$ ,  $df_1 = 2$ ,  $df_2 = 597$ ,  $p < .05$ ) based on students’ academic ability is significant at 0.05 level of significant. A multiple comparison analysis was carried out using Scheffe post hoc. This was done in order to carry out a pair wise comparison of the academic groups exhibited response aberrance. The result is presented in Table 5

**Table 5: Scheffe Multiple Comparison of Students’ Response Aberrance as Measured by W\* and B\* Based on Academic Ability**

Dependent Variable	(I) Students’ Ability	(J) Students’ Ability	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
W*	Low	Average	.369409*	.130267	.019	.04893	.68989
		High	-1.116773*	.168326	.000	-1.53088	-.70267
	Average	Low	-.369409*	.130267	.019	-.68989	-.04893
		High	-1.486182*	.156159	.000	-1.87036	-1.10201
	High	Low	1.116773*	.168326	.000	.70267	1.53088
		Average	1.486182*	.156159	.000	1.10201	1.87036
B*	Low	Average	-.516997	.385927	.409	-1.46647	.43247
		High	1.070914	.500911	.104	-.16144	2.30327
	Average	Low	.516997	.385927	.409	-.43247	1.46647
		High	1.587911*	.464229	.003	.44580	2.73002
	High	Low	-1.070914	.500911	.104	-2.30327	.16144
		Average	-1.587911*	.464229	.003	-2.73002	-.44580

\*. The mean difference is significant at the 0.05 level.

The pair wise comparison of the three academic ability groups as contained in Table 5 showed that there is significant differences in W\* and B\* mean values. This implies a significant difference in students response aberrance measured by each of W\* and B\*. Table 5 showed that a pairwise comparison of low academic ability students’ W\* value with each of average and high ability students showed a significant difference with an indication that while the response aberrance exhibited by lower ability students is significantly higher than that of the average ability students, the response aberrance exhibited by the high ability students as measured by W\* is significantly higher than that of low ability students. Table 5 also showed that only the high and average ability students pairwise comparison of the B\* value showed a significant difference with an indication that the response aberrance of average academic ability students as measured by B\* is significantly higher than that of high academic ability students. Therefore, with the comparison in Table 5, it is obvious that the response aberrance of high ability students is more than that of low and average ability as measured by W\*. This showed that the examinees missed some of the questions within their ability level. Also, the response aberrance as measured by B\* for average ability examinees is more than that of high and low ability students which indicates that some students responded to some items that are beyond their ability level.

## ***Discussions***

The test conducted in this study was given under stringent time pressure. The results of last school exam in Mathematics were used to classify students into low, average and high ability students. Response aberrance for the response of all students in the study was computed with the use of MATLAB. The values of response aberrance as measured by  $W^*$  and  $B^*$  were used for the analysis in this study. The values gotten after computation of response of students that took part in the test revealed that there is response aberrance in Mathematics among secondary school students in Osun State. The result indicates that the students missed some of the items they are expected to get correctly and responded correctly to difficult items they are expected to miss. This may be due to careless mistakes, lucky guessing, plodding, and anxiety etc. This is in agreement with the findings of Edith and Joop (1998), if a student has missed the easiest questions (items) and responded very well to the difficult and the most difficult items, the response pattern is quite unusual, there is aberrance. It thus appears that students' scores in a test may not show true ability of such students because a student may guess right and pass the test.

In comparing students' response aberrance in Mathematics based on Mathematical ability, it was discovered that high ability students exhibited greater response aberrance than low and average ability students as measured by  $W^*$ . This may be due to careless mistakes, inadequate preparation and start up anxiety. This result contradicts the findings of Chen (2004) which reported that low ability students demonstrated higher response aberrance than high and medium ability students as measured by  $W^*$ . The poor responds of low ability students to easy items according to the finding of Chen (2004) may be caused by their ability, poor preparation and plodding. Also, the result of this study showed that average ability students exhibited greater response aberrance than low and high ability students as measured by  $B^*$ . When students are desperate to pass a test with the items that are beyond their ability, they may involve in lucky guessing, cheating if it is possible in order to pass. But in contrast, Chen(2004) reported that low ability students exhibited greater response aberrance as measured by  $B^*$ . The reason for the disparity may be the contents of the items in the test and the time allowed for completing the test.

## ***Conclusion***

This study provided information on the prevalence and effect of mathematical ability on response aberrance in Mathematics. It also showed that there are differences in students' response aberrance in Mathematics among secondary school students. It is therefore suggested that teachers should as much as possible reduce the rate at which response aberrance occur by ensuring that students ability is considered in constructing test items.

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