Movement of Rocket and Its Impact on Unguided Rocket Trajectory[1]

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Introduction

Literature dealing with the issues of unguided rocket movement, usually resolve movement of rockets intended for ground firing from rocket launchers, for which, their motion relative to the earth is assumed. However, unguided rockets are fired from planes, tanks and armored personnel carriers during movement etc… Then they are fired from rocket pod that is in motion and by this movement rocket pod affects rocket at the time when it is in contact with rocket. The initial impulse which rocket gives to rocket pod is also reflected in further movement of rocket in the active section of trajectory.

Addressing the impact of moving rocket pod to unguided rocket trajectory, we use the method that is described in the literature [6], to derive motion equations for unguided rockets fired from fixed rocket pod. In [6] has been shown that the descending phase of rocket movement from rocket pod may have major impact on rocket movement direction, which, already in the early stage, may differ appreciably from direction given by rocket pod. Therefore, significant influence of rocket movement can be expected if rocket pod is in in motion.

For the determination of external forces impact on rocket movement in [6] is rocket trajectory divided into four sections. The first section is movement of rocket in rocket pod, the second begins at the moment when center of gravity of rocket leaves the rocket pod and ends after contact of rocket with rocket pod is interrupted. At this point begins the third section of trajectory, which border with the fourth section is the point when propellant charge burns out. Rocket movement in the third and fourth trajectory section is no longer directly affected by rocket pod and therefore the equations for these two trajectory sections will remain unchanged. Changes in the system of equations will occur only in the first two sections of trajectory the and therefore these two sections are subjects of further analysis.

Basic assumptions and general relations

In order to simplify solution, we exclude various spurious and false influences from our consideration. We will study the problem in normal weather conditions (also in windlessness) and assuming an axially symmetrical rocket. We therefore assume that rocket center of gravity is on its geometrical axis, in this axis lies also the thrust vector of a rocket motor and vector of environment resistance lies in a plane of resistance determined by rocket axis and speed vector of rocket center of gravity. It is further believed that the aerodynamic moment is independent from rocket rotation around its longitudinal axis and depends only on speed of movement, incidence angle and characteristics of atmosphere.

We assume, that rocket pod is so rigid, that there is no deformation and its assumed movement is not affected by rocket reverse action of force. All rocket movements are known.

[1] In this article, equations for advanced and rotational movement of unguided rocket for its descending from rocket pod, which is moving, are derived. The possibilities of their practical use are also mentioned.

We will use a total of five rectangular rightward rotating coordinate systems for deriving the equations. The first coordinate and basic system is inertial system (x, y, z), which is attached to the ground surface. The x-axis is horizontal in firing direction, the y-axis is vertical and the z-axis is horizontal to the right of firing direction. Vertical plane (x, y) intersecting the longitudinal axis of the rocket and the axis of rocket pod in the beginning of rocket movement. In this coordinate system we will express gradual movement of rocket center of gravity, as well as all rotary movements.

The second coordinate system (x1, y1, z1) has an axis x1 permanently in the axis of rocket, axis y1 is in a vertical plane through the axis x1 and thus the axis y and axis z1 is thus in a horizontal plane (x, z). Position of system (x_1, y_1, z_1) in the system (x, y, z) is given by angles α_1 and γ_1. Angle α_1 is the angle between axis of the rocket and horizontal plane (x, z) and between axes y and y_1 and angle γ_1 is the angle between vertical planes (x, y) and (x_1, y_1). It is also the angle between axes z and z_1. Relative position of these two systems, shown in Fig. 1, also applies to relative position of the other two systems in regard to the inertial coordinate system.

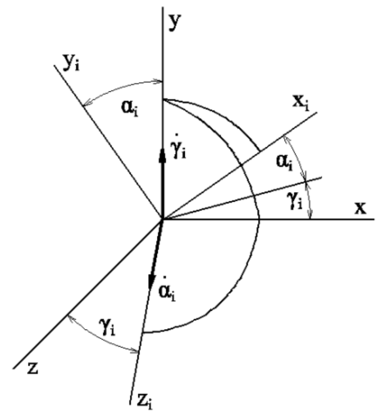

Fig. 1: Position of coordinate systems with respect to inertial coordinate system

The third coordinate system (x_2, y_2, z_2) has the axis x_2 in the direction of velocity vector and axis y_2 in a vertical plane passing through the axis x_2. The axis z_2 is in horizontal plane (x, z). Position of this system is defined in an inertial coordinate system by angles α_2 and γ_2. Their determination is similar as for coordinate system (x_1, y_1, z_1) – fig. 1.

The fourth coordinate system is system (x_3, y_3, z_3), which axis x_3 is in rocket pod axis, y_3, again in the vertical plane through the axis x_3 and axis z_3 is horizontal. Position of this system is determined in coordinate system (x, y, z) similarly as in both previous systems, namely by angles α_3 and γ_3.

Last fifth coordinate system (ξ, η, ζ) is attached to the rocket and carries out all movements with it. Axis ξ lies in longitudinal axis of rocket, axes η and ζ are perpendicular to it. By angle φ we express indexing of rocket from vertical plane. It is the angle between the axis η and y_1 or between axes ζ and z_1.

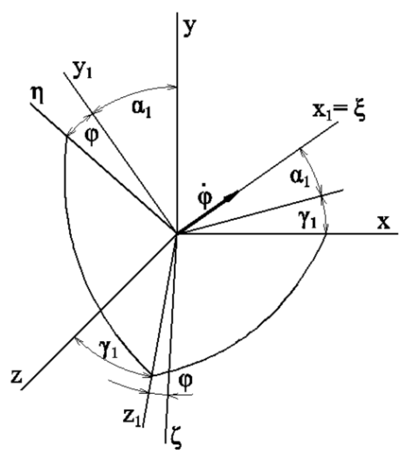

Fig. 2: Mutual position of systems (ξ, η, ζ) and (x_1, y_1, z_1)

For transformation of the individual constants in coordinate system (x_j, y_j, z_j) when i = 1, 2 and 3 to inertial coordinate system (x, y, z), we use equation

$$X = A_i X_i \tag{1}$$

And for transformation between coordinate systems (ξ, η, ζ) and (x_1, y_1, z_1), equation

$$X_1 = A \tag{2}$$

In these equation

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} X_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \beth = \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix}$$

$$A_1 = \begin{bmatrix} \cos\alpha_i \cos\gamma_i & -\sin\alpha_i \cos\gamma_i & \sin\gamma_i \\ \sin\alpha_i & \cos\alpha_i & 0 \\ -\cos\alpha_i \sin\gamma_i & \sin\alpha_i \sin\gamma_i & \cos\gamma_i \end{bmatrix} \tag{3}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \tag{4}$$

The matrices (3) and (4) are orthogonal (see [5] chap. 5.10), therefore, for inverse transformation applies:

$$X_i = A'_i X \tag{5}$$

$$\nexists = A' A_i \tag{6}$$

Where A'_i and A' are transposed matrices to matrices A_i and A. If we need to transform elements between any coordinate system X_i and X_j, we use equations (1) and (5):

$$X_i = A'_i X \qquad\qquad X = A_j X_j$$

$$X_i = A_{ij} X_j$$

$$A_{ij} = A'_i A_j \tag{7}$$

After multiplication of matrices, we get:

$$A_1 = \begin{bmatrix} \begin{matrix} sin\alpha_i sin\alpha_j \\ +cos\alpha_i cos\alpha_j cos(\gamma_i - \gamma_j) \end{matrix} & \begin{matrix} sin\alpha_i cos\alpha_j \\ -cos\alpha_i sin\alpha_j cos(\gamma_i - \gamma_j) \end{matrix} & -cos\alpha_i sin(\gamma_i - \gamma_j) \\ \begin{matrix} cos\alpha_i sin\alpha_j \\ -sin\alpha_i cos\alpha_j cos(\gamma_i - \gamma_j) \end{matrix} & \begin{matrix} cos\alpha_i cos\alpha_j \\ +sin\alpha_i sin\alpha_j cos(\gamma_i - \gamma_j) \end{matrix} & sin\alpha_i sin(\gamma_i - \gamma_j) \\ cos\alpha_i sin(\gamma_i - \gamma_j) & -sin\alpha_j sin(\gamma_i - \gamma_j) & cos(\gamma_i - \gamma_j) \end{bmatrix} \tag{8}$$

For calculation of transformation matrix with coordinate systems (ξ, η, ζ) and (x, y, z) we use equations (5) and (6), from which follows:

$$\nexists = A' A_1 X = B' X \tag{9}$$

$$B' = A' A'_1$$

$$A_1 = \begin{bmatrix} \begin{matrix} cos\alpha_1 cos\gamma_1 \\ +cos\alpha_i cos\alpha_j cos(\gamma_i - \gamma_j) \end{matrix} & \begin{matrix} sin\alpha_1 \\ -cos\alpha_i sin\alpha_j cos(\gamma_i - \gamma_j) \end{matrix} & -cos\alpha_1 sin\gamma_1 \\ \begin{matrix} cos\alpha_i sin\alpha_j \\ -sin\alpha_i cos\alpha_j cos(\gamma_i - \gamma_j) \end{matrix} & \begin{matrix} cos\alpha_i cos\alpha_j \\ +sin\alpha_i sin\alpha_j cos(\gamma_i - \gamma_j) \end{matrix} & sin\alpha_i sin(\gamma_i - \gamma_j) \\ cos\alpha_i sin(\gamma_i - \gamma_j) & -sin\alpha_j sin(\gamma_i - \gamma_j) & cos(\gamma_i - \gamma_j) \end{bmatrix} \tag{10}$$

Matrices A_{ij} and B' are equally orthogonal, so we cannot operate with them.

Equations for the first part of rocket trajectory

A. Gradual movement

In the first part of rocket trajectory is rocket center of gravity moving in rocket pod area. Gradual movement of inertial coordinate system is given by vector sum of relative movement of rocket with respect to rocket pod and its movement. For our solution is particularly necessary to know the speed of rocket in this section at the moment when rocket center of gravity is at the end of rocket pod. Suppose we know the speed of rocket pod muzzle v_u and components of that speed v_{uy} and v_{uz} in axes x, y, z. Because of rocket pod, rocket center of gravity is moving in direction of its axes i.e. in our coordinate systems in direction of axis x_3. Rocket relative speed in regard to rocket pod v_r, has therefore components in axes y_3 and z_3 equal to zero.

Sought rocket relative speed at the end of rocket pod v_{jr} can be calculated by integrating equation

$$a_\xi = a_F(t) - g sin\alpha_3(t) - a_{FR} \qquad (11)$$

for trajectory from the start of rocket center of gravity movement in regard to rocket pod, to the moment, when rocket center of gravity reaches the end point of rocket pod.
In equation (11) is

a_ξ – acceleration of rocket in regard to rocket pod,
a_F (t) – acceleration of rocket engine thrust,
g – acceleration of gravity,
α_3 (t) – angle between rocket pod axis and horizontal plane,
a_{FR} – delay caused by friction of rocket in rocket pod.

The components of rocket relative speed at the end of rocket pod in the coordinate system (x_3, y_3, z_3) are thus (v_{ir}, 0, 0). For their transformation to basic system (x, y, z) are used equations (1) and (3):

$$\begin{aligned} v_{irx} &= v_{ir} cos\alpha_3 cos\gamma_3 \\ v_{1ry} &= v_1 sin\alpha_3 \\ v_{1rz} &= -v_{1r} cos\alpha_3 sin\gamma_3 \end{aligned} \qquad (12)$$

Components of speed v_1 in inertial system at the moment when rocket center of gravity is at the end of rocket pod are:

$$\begin{aligned} v_{1x} &= v_{1rx} + v_{ux} \\ v_{1y} &= v_{1ry} + v_{uy} \\ v_{1z} &= v_{1rz} + v_{uz} \end{aligned} \qquad (13)$$

and speed itself is:

$$v_1 = \sqrt{v_{1x}^2 + v_{1y}^2 + v_{1z}^2} \qquad (14)$$

Position of point, at which rocket center of gravity leaves rocket pod, is in inertial system given by position of rocket pod muzzle at given moment, thus, it is given by relation, which defines movement of rocket pod.

B. Rotational movement

In the first part of rocket trajectory are rotational movements of rocket around the transverse axes given by movements of rocket pod. Then applies:

$$\dot{\alpha}_3 = \dot{\alpha}_1 \qquad \dot{\gamma}_3 = \dot{\gamma}_1$$

Rocket rotation around its longitudinal axis can however be obtained either by component of rocket motor thrust, which vector is skew to rocket longitudinal axis or helical rocket bearing in rocket pod. In the first case we gain operating speed at the end of the first section of trajectory by solving the equation

$$\ddot{\varphi} = \frac{M_F}{C} \qquad (15)$$

where M_F is moment of rocket motor thrust in regard to longitudinal axis of rocket and C is moment of rocket inertia to this axis. If rocket operating speed is forced by helical rocket bearing in rocket pod, angular velocity at the end rocket pod depends on rocket relative speed v_{1r} in regard to rocket pod and convolution rising δ for position, at which is rocket center of gravity at the end of rocket pod:

$$\dot{\varphi}_1 = \frac{2v_{1r} tg\delta}{d} \qquad (16)$$

Equations for the second part of rocket trajectory

The second part of rocket trajectory starts at the moment, when rocket center of gravity is at the end of rocket pod. In this situation, components of rocket muzzle and rocket center of gravity speeds in direction of axes y_3 and z_3 are identical. Sooner or later, this identity breaks due to action of aerodynamic forces on a rocket, due to changes in speed and direction of rocket pod movement, due to action of gravity on a rocket, etc... For rocket and rocket pod interactions, point of rocket contact with rocket pod and speed v_k of this point is crucial. Speed v_k is given not only by speed of rocket center of gravity, but also by angular speed of rocket around its center of gravity.

From the difference of rocket pod muzzle components speed v_u and speed of that point of rocket, which is in contact with rocket pod v_k, in the direction of axes y_3 and z_3, we determine acceleration, by which is rocket affected in its movement through rocket pod. Accelerations thus have, in directions of this referred-to axes accelerations:

$$a_{py3} = \frac{v_{uy3} - v_{ky3}}{\Delta t} \tag{17}$$

$$\Delta_{pz3} = \frac{v_{uz3} - v_{kz3}}{\Delta t} \tag{18}$$

With emergence of acceleration affecting movement of rocket, is also formed the moment affecting rotational rocket movements, which components in axes y_3 and z_3 are:

$$\begin{aligned} M_{py3} &= a_{pz3} m(t) l(t) \\ M_{pz3} &= a_{py3} m(t) l(t) \end{aligned} \tag{19}$$

Once again, we suggest, that retroaction of rocket on rocket pod is neglected. In equations (19) is m(t) the weight of rocket at the moment and l(t) is rocket center of gravity distance from rocket pod cross-section at point, in which is rocket in contact with rocket pod. For calculation of acceleration in the direction of axes y_3 and z_3, is necessary to transform components of rocket pod muzzle speed from coordinate system (x, y, z) to coordinate system (x_3, y_3, z_3) according to equation (5):

$$\begin{aligned} v_{uy3} &= -v_{ux} sin\alpha_3 cos\gamma_3 + v_{uy} cos\alpha_3 + v_{uy} sin\alpha_3 sin\gamma_3 \\ v_{uz3} &= v_{ux} sin\gamma_3 + v_{uz} cos\gamma_3 \end{aligned} \tag{20}$$

Speed components of contact point of rocket with rocket pod in axes y_3 and z_3 consist of the components - rocket center of gravity gradual movement and corresponding circumferential speed due to rotational movements of rocket:

$$\begin{aligned} v_{ky3} &= v_{y3} - \omega_{z3} l(t) \\ v_{kz3} &= v_{z3} + \omega_{y3} l(t) \end{aligned} \tag{21}$$

We obtain the components of rocket gradual speed v_{y3} and v_{z3} by transformation, using equations (7) and (8) from coordinate system (x_2, y_2, z_2), in which, the vector of rocket gradual speed has coordinates (v, 0, 0):

$$\begin{aligned} v_{y3} &= v[cos\alpha_3 sin\alpha_2 - sin\alpha_3 cos\alpha_2 cos(\gamma_3 - \gamma_2)] \\ v_{z3} &= v[cos\alpha_2 sin(\gamma_3 - \gamma_2)] \end{aligned} \tag{22}$$

Components of angular speed of rocket axis in axes y_3 and z_3 are obtained from angular speeds α_1 and γ_1, which are one of the results of basic equations solution and their components in the coordinate system (x, y, z) are according to Fig. 1:

$$\begin{aligned} \omega_x &= \dot{\alpha}_1 sin\gamma_1 \\ \omega_y &= \dot{\gamma}_1 \\ \omega_z &= \dot{\alpha}_1 cos\gamma_1 \end{aligned} \tag{23}$$

By their transformation to coordinate system (x_3, y_3, z_3) and by using equations (1) and (3) we obtain:

$$\omega_{y3} = \dot{\alpha}_1 sin\alpha_3(\gamma_3 - \gamma_1) + \gamma_1 c\dot{o}s\alpha_3$$
$$\omega_{z3} = \dot{\alpha}_1 cos(\gamma_3 - \gamma_1) \quad (24)$$

Accelerations caused by movement of rocket pod and expressed by equations (17) and (18), for calculation of which are used equations (21) – (24), are transformed to inertial coordinate system (x, y, z) and by that, we obtain final equations which express the effect of rocket pod moving on gradual movement of rocket center of gravity. We use equations (1) and (3) for transformation:

$$\begin{aligned} a_{px} &= -a_{py3} sin\alpha_3 cos\gamma_3 + a_{pz3} sin\gamma_3 \\ a_{py} &= a_{py3} cos\alpha_3 \\ a_{pz} &= a_{py3} sin\alpha_3 sin\gamma_3 + a_{pz3} cos\gamma_3 \end{aligned} \quad (25)$$

While solving rotational movements of rocket, we use, according to [6] Euler´s equations. For this task, is necessary to transform moments generated by movement of rocket pod and expressed by equations (19) to coordinate system of rocket body (ξ, η, ζ).

According to equations, (6) and (7) applies:

$$\begin{aligned} \in &= A' X_1 \\ X_1 &= A_{13} X_3 \\ \in &= A' A_{13} X_3 \end{aligned} \quad (26)$$

Conjunction of matrices A' A_{13} according to relations (4) and (8) has form:

$$A' A_{13} = \begin{matrix} sin\alpha_1 sin\alpha_3 + & sin\alpha_1 cos\alpha_3 + & -cos\alpha_1 sin(\gamma_1 - \gamma_3) \\ +cos\alpha_1 cos\alpha_3 cos(\gamma_1 - \gamma_3) & -cos\alpha_1 sin\alpha_3 cos(\gamma_1 - \gamma_3) & \\ cos\varphi[cos\alpha_1 sin\alpha_3 + & cos\varphi[cos\alpha_1 cos\alpha_3 + & sin\alpha_1 sin(\gamma_1 - \gamma_3) cos\varphi \\ -sin\alpha_1 cos\alpha_3 cos(\gamma_1 - \gamma_3)] & +sin\alpha_1 sin\alpha_3 cos(\gamma_1 - \gamma_3)] & +cos(\gamma_1 - \gamma_3) sin\varphi \\ +cos\alpha_3 sin(\gamma_1 - \gamma_3) sin\varphi & -sin\alpha_3 sin(\gamma_1 - \gamma_3) sin\varphi & \\ sin\varphi[-cos\alpha_1 sin\alpha_3 + & -sin\varphi[cos\alpha_1 cos\alpha_3 + & -sin\alpha_1 sin(\gamma_1 - \gamma_3) sin\varphi \\ +sin\alpha_1 cos\alpha_3 cos(\gamma_1 - \gamma_3)] & +sin\alpha_1 sin\alpha_3 cos(\gamma_1 - \gamma_3)] & +cos(\gamma_1 - \gamma_3) cos\varphi \\ +cos\alpha_3 sin(\gamma_1 - \gamma_3) cos\varphi & -sin\alpha_3 sin(\gamma_1 - \gamma_3) cos\varphi & \end{matrix} \quad (27)$$

According to equation (26) and by using matrix (27), we obtain components of moments which arose during moving of rocket pod and affect rotational moving of rocket in coordinate system (ξ, η, ζ)

$$M_{P\xi} = M_{py3}[sin\alpha_1 cos\alpha_3 - cos\alpha_1 sin\alpha_3 cos(\gamma_1 - \gamma_3)] - M_{pz3} cos\alpha_1 sin(\gamma_1 - \gamma_3),$$
$$M_{p\eta} = M_{py3}\{cos\varphi[cos\alpha_1 cos\alpha_3 + sin\alpha_1 sin\alpha_3 cos(\gamma_1 - \gamma_3)] - sin\alpha_3 sin(\gamma_1 - \gamma_3) sin\varphi\} + M_{pz3}[sin\alpha_1 sin(\gamma_1 - \gamma_3) cos\varphi + cos(\gamma_1 - \gamma_3) sin\varphi] \quad (28)$$
$$M_P^{\xi} = -M_{py3}\{sin\varphi[cos\alpha_1 cos\alpha_3 - sin\alpha_1 sin\alpha_3 cos(\gamma_1 - \gamma_3)] + sin\alpha_3 sin(\gamma_1 - \gamma_3) cos\varphi\} + M_{pz3}[cos(\gamma_1 - \gamma_3) cos\varphi - sin\alpha_1 sin(\gamma_1 - \gamma_3) sin\varphi]$$

In deriving equations for rotational movement of unguided rocket for motionless rocket pod were in [6] chapter 4.2 of derived equations (4.29) in this form:

$$\begin{aligned} M_\xi &= M_F \\ M_\eta &= M_G sin\varphi + M_R cos(\varepsilon - \varphi) \\ M_\zeta &= M_G cos\varphi + M_R sin(\varepsilon - \varphi) \end{aligned} \quad (29)$$

In these equations M_F is moment causing rotation around longitudinal axis of rocket by effect of rocket motor thrust component. M_G is moment generated during descend of rocket from rocket pod by action of gravity and M_R is moment generated by rocket axis, speed vector and axis z_1. If rocket pod is moving during descend of rocket, equation (29) will be extended to include relations (28) expressing the effect of this movement:

$$\begin{aligned} M_\xi &= M_F + M_{p\xi} \\ M_\eta &= M_G sin\varphi + M_R cos(\varepsilon - \varphi) + M_{p\xi} \\ M_\zeta &= M_G cos\varphi + M_R sin(\varepsilon - \varphi) + M_{p\xi} \end{aligned} \quad (30)$$

After substituting these moments to Euler´s equations for rotational movement and after adjustments used in [6], chapter 4.2, we get the equations in following form:

$$\begin{aligned} M_{FR} + M_{p\xi} &= C(\ddot{\varphi} + \ddot{\gamma}_1 sin\alpha_1 + \dot{\alpha}_1\dot{\gamma}_1 cos\alpha_1) \\ M_G sin\varphi + M_R cos(\varepsilon - \varphi) + M_{p\eta} &= Ecos\varphi + Fsin\varphi \\ M_G cos\varphi + M_R sin(\varepsilon - \varphi) + M_{p\xi} &= Esin\varphi + \mathrm{F}cos\varphi \end{aligned} \quad (31)$$

In equations (31)

$$E = A\ddot{\gamma}_1 cos\alpha_1 - (2A - C)\dot{\alpha}_1\dot{\gamma}_1 sin\alpha_1 + C\alpha_1\dot{\varphi}$$

$$F = A\ddot{\alpha}_1 + (A - C)\dot{\gamma}_1^2 sin\alpha_1 cos\alpha_1 - C\dot{\gamma}_1\dot{\varphi}\cos\alpha_1$$

In these relations, A is moment of rocket inertia to its transverse axis. By multiplying the second equation of system (31) sin φ and the third equation cos γ with collateral substitution from system of equations (30) we get:

$$M_G + M_R sin\varepsilon - M_{py3} sin\alpha_3 sin(\gamma_1 - \gamma_3) + M_{pz3} cos(\gamma_1 - \gamma_3) = F \quad (32)$$

Similarly, by multiplication of the second equation cosφ and the third equation (-sinφ), we obtain next equation:

$$M_R cos\varepsilon + M_{py3}[cos\alpha_1 cos\alpha_3 + cos\alpha_1 sin\alpha_3 cos(\gamma_1 - \gamma_3)] + M_{pz3} sin\alpha_1 sin(\gamma_1 - \gamma_3) = E \quad (33)$$

From the first equation of the system (31) and equations (32) and (33), using relations for E and F, we express, after modifications, the acceleration of rotational movements of unguided rocket on the second section of its trajectory in this form:

$$\varphi = \frac{M_F + M_{p\xi}}{C} - \gamma_1 sin\alpha_1 - \alpha_1\gamma_1 cos\alpha_1 \quad (34)$$

$$\ddot{\alpha}_1 = \frac{1}{A}\left[M_G - M_R sin\varepsilon + M_{py3} sin\alpha_3 sin(\gamma_1 - \gamma_3) - M_{pz3} cos(\gamma_1 - \gamma_3) \pm (A - C)\dot{\gamma}_1^2 sin\alpha_1 cos\alpha_1 + C\dot{\varphi}\dot{\gamma}_1 cos\alpha_1\right] \quad (35)$$

$$\ddot{\gamma}_1 = \frac{1}{Acos\alpha_1}\frac{1}{A}\left[M_{py3}\left(cos\alpha_3 + sin\alpha_3 co\quad(\gamma_1 - \gamma_3) + M_{pz3} tg\alpha_1 sin(\gamma_1 - \gamma_3) + \quad (2A - C)\dot{\alpha}_1\dot{\gamma}_1\right)\right] \quad (36)$$

Equations (34) - (36) are therefore fundamental equations that are used to solve rotational movement of the rocket on the second section of trajectory. Equation (34) expresses angular acceleration of rocket around its longitudinal axis, equation (35) angular acceleration of axis of rotation of rocket around cross-horizontal axis of rocket and equation (36) is used to calculate the angular acceleration of vertical plane passing through the axis of rotation of rocket around vertical line passing through rocket center of gravity, i.e. around vertical line parallel to the y-axis of inertial coordinate system.

Integrating the equations (34) - (36) gives us angular speeds and changes of position of rocket in inertial coordinate system on the second section of trajectory.

Effect of rocket pod movement on gradual movement of rocket center of gravity on given section of trajectory is expressed by equations (25). Motion equations of rocket center of gravity in this section of trajectory in inertial coordinate system, considering effect of rocket motor thrust, environmental resistance, gravity and rocket pod movement, have in regard to equations (4.21) in [6] this form:

$$\begin{aligned}
\frac{d^2x}{dt^2} &= a_1 cos\alpha_1 cos\gamma_1 - a_R cos\alpha_3 cos\gamma_3 - a_{py3} sin\alpha_3 cos\gamma_3 + a_{pz3} sin\gamma_3 \\
\frac{d^2y}{dt^2} &= a_F sin\alpha_1 - a_R sin\alpha_3 - g + a_{pz3} cos\alpha_3 \\
\frac{d^2z}{dt^2} &= -a_F cos\alpha_1 sin\gamma_1 + a_R cos\alpha_3 sin\gamma_3 + a_{py3} sin\alpha_3 sin\gamma_3 + a_{pz3} cos\gamma_3
\end{aligned} \tag{37}$$

Conclusion

Derivation of equations for expression of the effect of moving rocket pod on trajectory of unguided rocket directly follows the equations derived in [6] for rocket movement while rocket pod is fixed. Used derivation method is identical. In comparison with equations in [6], there are changes, due to effect of rocket pod movement, only in sections of trajectory, when rocket is in contact with rocket pod i.e. according to distribution mentioned in text, only in first two sections of trajectory. The basic equation for solving gradual movement of rocket with considering effect of moving rocket pod are expressed by equations (11) - (13) and (37), and for solving rotational movements of rocket under the same conditions by equations (15), if necessary (16) and equations (34) - (36) from this article. In mathematical expression of rocket movement in other sections of trajectory, changes due to rocket pod movement are no longer occurring and equations derived in [6] are fully usable.

With consideration of this fact, overall character of rotating and non-rotating rocket movements in active section of trajectory described in detail in appendix of this work [6], remains unchanged. The nature of this movement is also evident from figures in article [9]. Compared to mentioned results, changes will be in amplitude of oscillating motions in rotating and non-rotating rocket, as a result of significant changes in the conditions of descent of rocket from rocket pod, there will be also changes in the direction of rockets.

By solving equations derived in this article, is possible to gain an objective view of the effect of rocket pod moving on trajectory of unguided rocket. For given type of rocket and rocket pod and given character of its effect, is possible to detect changes in amplitude of oscillation motion of rocket, also changes in its movement and is possible to consider conditions for rocket pod movement, for which are changes in character of movement of rocket are no longer acceptable in light of practical use. Furthermore, equations can be used for detection of anomalies in rocket pod movement to dispersion of rocket hits, they can be used for determination of proper length of rocket pod etc... Derived equations have fairly wide practical application for given particular type of rocket and rocket pod.

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