Einstein's Relativity as an a Priori Theory from the Perspective of Kantian Philosophy

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Abstract

Several points made by Immanuel Kant in Critique of Pure Reason regarding a priori synthetic and a priori analytic theories relating to geometry and mathematics are discussed. These points are analyzed in regard to Einstein's Theories of Special and General Relativity, with analysis of the nature of space and time in a Riemannian geometry. The points of view of Einstein's contemporaries Lorentz, Minkowski, and Reichenbach are discussed. Perspectives from modern philosophers including Frege and Wittgenstein are analyzed. The argument is made that although there are perhaps some reasons to classify relativity theory as synthetic a priori, relativity theory is an analytic a priori theory, due to its essential geometrical and mathematical character, and derivation based on intuition. Implications for theoretical physics and a theory of everything are discussed.

Introduction

Immanuel Kant, notable enlightenment philosopher of the nineteenth century, made significant contributions to the foundation of modern philosophy, or metaphysics. In his Critique of Pure Reason [1], he elevated philosophy to the utmost level of rigor, far beyond a speculative endeavor, in a work based on sound reasoning rivaling that of mathematical proofs. His view of the sciences, in particular, math and physics, is noteworthy, because his philosophical reasoning reached conclusions about those fields which would agree with many of the notable figures in those fields. Kant's view of mathematics was that it was composed of a priori judgments, whose validity is necessary. For example, consider the axioms of geometry: they are not empirical concepts, but concepts based on recourse to intuition about space. They are moreover analytic a priori judgments, a categorization which describes mathematics as well. His view of physics, contrary to many philosophers, is that it is too composed of a priori judgments. They are a priori, based on intuition, but are synthetical as well, based on a synthesis of empirical propositions. Although modern science is viewed as an empirical pursuit, among the sciences, physics has a uniquely mathematical character. The central question of this work is this: What is the classification of an inherently mathematical theory of physics, namely general relativity, in the Kantian categorization of science? More explicitly, is it an analytic a priori theory or a synthetic a priori theory? Considering this question will help to clarify the geometrical nature of the theory of relativity. This is of timely relevance in the light of recent discoveries relating to general relativity from LIGO [2].

I contend that the view of a theory such as Einstein's theory of general relativity, with such elegance and sophistication, couldn't be the result of mere empirical judgments. That it must be for such a law of physics to necessarily reflect fundamental axioms of space and time. As we will see, according to Kant, space and time are intuitive concepts, and judgments about space and time are of an a priori nature. Kant, in his time, could only reflect upon the Newtonian physics of his day. A fundamental theory, such as special relativity, with its basis in the curved geometry of space and time, is necessarily a priori. At the same time, it is synthetic, because the deviations from Newtonian physics were observed experimentally and what was required was a more fundamental theory to replace it. While the theory can be shown to be a priori, does it follow from intuition? In the Newtonian system, which relies on Euclidean geometry, the mathematical basis is analytic a priori, for Euclidean geometry can be derived from intuition. The Galilean transformation of velocities, which follow experience but clearly needed empirical evidence, are clearly synthetic a priori. Special relativity, on the other hand, was based on non-Euclidean geometry. The relativistic system of Einstein, which relies on non-Euclidean geometry, the classification of the mathematics may be disputed.

Is the non-Euclidean geometry a priori, and if so, analytic or synthetic? Although non-Euclidean geometry is based on mathematical axioms, they were derived with a view to describing different phenomena that could not be explained by Euclidean geometry, and since it is not obvious how they could be derived from recourse to intuition, such a geometry could be classified as synthetic a priori. The Lorentz transformation of velocities was derived as a mathematical structure for electromagnetism. The Minkowski metric of space time was also conceived as a mathematical structure. These follow as synthetic a priori statements, as becoming mathematics not based on intuition, but based on a description of real-ity. If relativity is to be viewed as following from mathematical axioms, then it would consist of analytic a priori statements. Non-Euclidean geometry of which relativity involves is clearly analytic a priori, based on a different set of axioms.

Is intuition simply inadequate as a basis for the modern developments in theoretical physics? From the one hand, they are based in mathematics, and they should be an- alytic a priori judgments. But man is born without innate intuition of the space time continuum, Lorentz transformation, and so on, and therefore the mathematical struc tures which describe these physical theories should be synthetic a priori judgments, which also characterizes the physical theory of relativity that they underpin. We will attempt a resolution of this apparent contradiction.

Modern mathematics has passed the point where axioms and theorems may be derived by recourse to intuition. Romanian geometry and complex analysis, require the finest intellects to find truth in matters which may not be within the realm of intuitive cognition. They are a priori judgments, being mathematical ones. In the Kantian system, they may be classified as synthetic, for in Kant's own words, "we ascribe a quality to an object which is not according to intuition." In complex analysis, for example, we ascribe extra numbers, an imaginary one such as $i = \sqrt{-1}$, in addition to a real number, to specify what a complex number should be. The number i is clearly not an object of intuition, but the use of it satisfies certain axioms in physics, including relativity. Hence in the Kantian system, much of modern mathematics could be viewed as system of synthetic a priori statements. It will be argued, however, that non-Euclidean geometry could be classified as analytic a priori. And in this case, relativity, which depends on such a geometry, could be interpreted as analytic a priori as well.

2 Considerations from Kant

Kant categorizes propositions as analytical or empirical, and makes the following state-ment regarding analytical propositions [1]:

But an empirical proposition cannot possess the qualities of necessity and absolute universality, which nevertheless are the characteristics of all geo- metric propositions. As to the first and only means to arrive at such cognitions, namely through mere conceptions, or intuitions a priori, it is quite clear from mere conceptions, no synthetical cognitions, but only analytical ones, can be obtained. In light of the previous quotation, the contradiction is resolved by considering what is meant of intuition. As beings in space and time, man conceives of space and time from pure intuition. From Kant's statements, intuition is not necessarily limited to that which is trivial or obvious, as would be the case of elementary geometry. Then we might conclude, that relativity, as a theory grounded in geometry of spacetime, should consist of analytic a priori statements.

From the discussion of the Transcendental Analytic in the Critique of Pure Reason: Now this completeness of a science cannot be accepted with confidence on the guarantee of a mere estimate of its existence in an aggregate formed only by means of an idea of the totality of the a priori cognition of the understanding, and through the thereby determined division of the concep-tions which form the said whole; only by means of their connection in a system. Pure understanding distinguishes itself not merely from everything empirical but also completely from all sensibility.

To the extent that relativity is complete, and it was developed through cognition of understanding, theoretical means, such as following the logic of the gedanken (thought) experiment. We may venture to suggest that Kant foresaw the notion of science such as physics into theoretical based on a priori intuition, and cognition and experimental, based on empirical judgment. Kant means by pure understanding, theoretical understanding divorced from understanding arrived at from the senses and also from understanding based on reduction of data. Relativity sought to resolve the contradictions of new experiments with the previous prevailing theories of physics, and thus was motivated by empirical considerations, yet the empirical considerations would not in themselves form a basis of pure understanding.

From the discussion of Transcendental Logic, we may explore whether relativity is better classified as synthetical a priori propositions [1]: This synthesis is pure when the diversity is not given empirically but a priori (as that in space and time... But the synthesis of a diversity is the first requisite for the production of a cognition, which, in its beginning, indeed, may be crude and confused, and therefore in need of analysis still, synthesis is that by which means alone the elements of our cognitions are collected and united into a certain content, consequently it is the first thing on which we must fix our attention, if we wish to investigate the origin of knowledge.

Thus, given that relativity relies upon reasoning and deductions from cognitions in space and time, this might indicate that relativity is a synthetical a priori theory. Synthesis, generally speaking, is, as we shall afterwards see, the mere operation of the imagination…. But to reduce this synthesis to conceptions is a function of the understanding, by means of which we attain to cognition, in the proper meaning of the term.

Note the use of the word imagination, which is in contrast to the use of intuition, for it involves unifying disparate elements of our cognitions. This notion distinguishes the synthetic from the analytic propositions. The resolution of the contradiction depends on the conception of the theory of relativity as a geometry in which case it is an analytic a priori theory or as a cognition based on judgments about diversity of phenomena of space and time, in which case it a synthetic a priori theory. We may note the dialectic character of a priori judgments in science: from one hand, they may be viewed as synthetic; from the other hand, analytic. While Kant mentions analytic judgments as geometrical, he states from another point of view that geometric reasoning and deductions may be viewed as synthetically. We may venture to apply the concept of the Transcendental Dialectic, developed for metaphysics, to physics proper.

We may proceed to a discussion of the transcendental aesthetic. The transcendental characterizes "all knowledge which is occupied not so much with objects as with the mode of our knowledge of objects in so far as this mode of knowledge is to be possible a priori." The aesthetic pertains to the objects perceived by the senses. Time and space in the Transcendental Aesthetic are things of sensibility. Yet time and space are a priori intuitive concepts, according to Kant. Properly, time and space are object of sensible intuition. They are not things in themselves. Although Kant's description of time is not inconsistent with physics, at least Newtonian physics, the intuition of ordinary experience is implied by Kant. On the other hand, the relativistic conception of space time is consistent with geometrical intuition of four-dimensional spacetime. Yet four-dimensional spacetime is not the realm of sensible intuition, but rather the a priori intuition of mathematical reasoning.

Kant further implies that the mind-independent world does not come located in a spatial or a temporal matrix. Rather, it is the mind that organizes this manifold of raw intuition, as he called it, spatially and temporally. This notion, which seems absurd in the light of the Euclidean 3-dimensional spatial world, seems to admit the possibility of relativistic spacetime. For this implies that space and time may have manifestly different form to a nonhuman, sentient being, since the product of sensuous intuition. And for if time and space are not things in themselves, what they properties are may be divorced from the human intuition.

We must consider the representation of space according to Kant:

1. Space is not a conception which has been derived from outward experiences ....; in like manner, in order that I may represent them not merely as without, of, and near to each other, but also in separate places, the representation of space cannot be borrowed from the relations of external phenomena through experience; but, on the contrary, this external experience is itself only possible through the said antecedent representation.
2. Space is then a necessary representation a priori. We never can imagine or make a representation to ourselves of the nonexistence of space.
3. Space is no discursive, or as we say, general conception of the relation of things, but a pure intuition. Space is essentially one· · · Hence it follows that an a priori intuition lies at the root of all our conceptions of space.

Regarding the first point, Kant posits that space is intuitive. As space is intuitive, it is amenable to mathematical propositions regarding it. Regarding the second point, this statement might disagree with predictions of physics, such as the Big Bang Theory, where space is imagined to have not existed before a point in time. Indeed, in the general theory of relativity, space might collapse in the presence of mass so dense, such as a black hole. Regarding the third point, as relativity is necessarily a theory based on space, its conclusions based upon intuition regarding space would necessarily be a priori, and moreover, analytic.

Furthermore, although Kant has much in common with Newtonian world view, he dismisses absolute reality of space and time in the Transcendental Aesthetic:

Those, however, who assert the absolute reality of space and time, whether they take it to be subsisting or only inhering, must themselves come into conflict with the principles of experience. For if they decide in favor of the first (which generally is the position of the mathematical investigators of nature), then they must assume two eternal and infinite self-subsisting non- entities (space and time), which are there (yet without there being anything actual) only in order to comprehend everything actual within themselves. In a sense, it may be interpreted that Kantian philosophy may be consistent with relative notions of space and time, which are dependent on intuition and thus related to the position of the observer.

The dialectic nature of propositions about cosmology is mentioned by Kant. As
Posy discusses [3], Kant's attack on transcendental realism is especially potent in the "Antinomy" section of the "Dialectic," where he attempts to show that the realist (but not the transcendental idealist) is committed to four pairs of mutually contradictory propositions.... The first commits the realist to both the finite and infinite extent of the world in space and in past time....Posy continues [3]:

But, I point out, the "purity" of mathematical intuition leads Kant to believe that all mathematical judgments are decidable. So I conclude that though Kant advocates an intuitionistic logic for empirical discourse, he favors a classical logic for mathematics.

In a sense, Kant subscribed to intuition, like Einstein, but favored a classical logic, as Einstein rejected some of the philosophical aspects of quantum theory.

3 Considerations from the Theory of Relativity

In addition to Albert Einstein, Hendrik Lorentz and Minkowski were essential to crucial elements of the theory of relativity. It was developed in response to deviations from Newtonian physics, but the theory had an essential mathematical character.

3.1 Einstein

Intuition plays a key role in general relativity, according to Einstein [4]: The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic under-standing of experience, can reach them.
That these conceptions could be arrived at by intuition, whereas they are clearly not intuitive at a basic level. It is certainly a priori, but whether analytic? Einstein himself called his theory analytic [5]:

But in addition to this most weighty group of theories, there is another group consisting of what I call theories of principle. These employ the analytic, not the synthetic method. Their starting-point and foundation are not hypothetical constituents, but empirically observed general properties of phenomena, principles from which mathematical formula are deduced of such a kind that they apply to every case which presents itself. Thermo-dynamics, for instance, starting from the fact that perpetual motion never occurs in ordinary experience, attempts to deduce from this, by analytic 6processes, a theory which will apply in every case. The merit of constructive theories is their comprehensiveness, adaptability, and clarity, that of the theories of principle, their logical perfection, and the security of their foundation. The theory of relativity is a theory of principle. To understand it, the principles on which it rests must be grasped. But before stating these it is necessary to point out that the theory of relativity is like a house with two separate stories, the special relativity theory and the general theory of relativity.

Einstein perhaps intends that a theory of principle, such as relativity, is a priori. And analytic in that the theory of relativity recourses to fundamentally observed principles as its basis, but the mathematical formula are deduced. It is not said, but implied elsewhere, that the basis of this deduction relies on intuition.

3.2 Lorentz

To paraphrase Hendrik Lorentz, Einstein's contemporary [6]: Einstein's theory has the highest degree of aesthetic merit : every lover of the beautiful must wish it to be true. It gives a vast unified survey of the operations of nature, with a technical simplicity in the critical assumptions which makes the wealth of deductions astonishing. It is a case of an ad-vance arrived at by pure theory: the whole effect of Einstein's work is to make physics more philosophical (in a good sense) and restore some of that intellectual unity which belonged to the great scientific

systems of the seventeenth and eighteenth centuries, but which was lost through increasing specialization and the overwhelming mass of detailed knowledge. As Lorentz wrote:

That Einstein's advance was from pure theory, it would be from a priori cognition in the Kantian terminology; the intellectual unity refers to the Kantian conception of cognitions collected and united into a synthetic a priori judgment. As the theory appeals to the sensibilities, it has an aesthetic quality, which relates to the Transcendental Aesthetic. This is appropriate, since the intuition of scientists is nonetheless sensuous intuition.

Lorentz also commented:

Einstein's work, we may now positively expect, will remain a monument of science; his theory entirely fulfills the first and principal demand that we may make, that of deducing the course of phenomena from certain principles exactly and to the smallest details.

The notion of deduction from first principles would tend to support the contention that relativity is a synthetic a priori theory. The idea of deduction would seem to imply that it is not the primary intuition, but a synthesis of intuitive conceptions. However, this runs counter to Einstein's own characterization of his theory as analytic.

3.3 Thought Experiments

Einstein developed his theory carefully with the aid of thought experiments hypothetical situations, which, when analyzed according to physical reasoning, would tend to lead to elucidation of the true theory. The situation of a spaceship accelerating in a gravitational–field free region was analyzed. The motion of an object inside the spaceship was analyzed. Based on reasoning from first principles in the hypothetical situation, the conclusion was reached that the consequences for the motion were no different than if the acceleration . Einstein consequently postulated the equivalence of inertial mass and gravitational mass. The analysis proceeds from a synthesis of a priori judgments, and hence, we have a situation in which the conclusion can be characterized as syntactical a priori. The so-called parable of the two travelers is another example of a thought experiment. Two travelers with mass travel in relative motion, and their observed paths are curved. The classical view would be that they are accelerating due to gravity; relativity holds that it is not gravity, but the curvature of spacetime which is responsible for their motion [7].

At the same time, the idea of experiment in thought experiment indicates a type of empirical reasoning. Thought experiments therefore have a dialectic quality on the one hand, empirical; on the other hand, deductive by intuition. Yet the intuitive reasoning based on space and time, according to Kant, must be analytic a priori.

3.4 Geometry of Spacetime

Special relativity is situated within the geometry of 4-dimensional spacetime. The Lorentz transformation of coordinates, velocities, etc., are based upon this space time geometry. Kant clearly describes space and time as distinct concepts based on understanding based on intuition, he could not have anticipated the unification of space and time into a geometry uniting them, which relativity requires. That light does not travel at infinite velocity, but at some finite value, irrespective of the frame of reference at which its velocity is measured, is a consequence of the space time geometry. A consequence of the finite speed of light is that events that are simultaneous in one reference frame may not be so in another reference frame. Such a conception is a priori–but whether synthetic or analytic? And does this formulation of space and time accord with intuition? These issues will be resolved in the next section. In fact, the Lorentz transformation was derived from consideration of the invariance of the speed of light, in the context of electromagnetic fields, and was thus a purely a priori mathematical conception divorced from empirical consideration. The transformation from the (x,y,z,t) to the moving (x_1,y_1,z_1,t_1) frame with a velocity v and relativistic boost γ can be written as follows [8]:

$x_1 = \gamma(x - vt)$
$y_1 = y$
$z_1 = z$
$t_1 = \gamma(t - vx/c^2)$.

A consequence of these transformations is that there is no longer simultaneity, i.e., there is no more absolute time. Of course, it is known that Kant subscribed to the notion of absolute time. However, this is in agreement with the Newtonian physics 8known at his time. Length contraction also describes a phenomenon in which space is not absolute.

In order to consider the curvature of spacetime, we need to consider General Relativity [9], the curvature of space time may be described by the Einstein field equations [8]:

$$8\pi G R_{\mu\nu} - R g_{\mu\nu} + \Lambda g_{\mu\nu} = 4\, T_{\mu\nu} \quad (3.2)$$

In equation (3.2), $R_{\mu\nu}$ denotes the Ricci tensor, g μν describes the metric tensor, T μν denotes the stress–energy tensor, and Λ parametrizes the cosmological constant. The fields T μν are inter–related with the curvature R μν and the metric tensor. It is a quite profound equation can be used to describe phenomena such as black holes. Further- more, the curvature of space time is consistent with Kant's Transcendental Aesthetic in the sense that it is an object of intuition. The metric tensor $g_{\mu\nu}$ describes the equation

$$ds^2 = c^2\, dt^2 - dx^2 - dy^2 - dz^2 = g_{\mu\nu}\, dx_\mu\, dx_\nu \quad (3.3)$$

The metric can be written as diag(1, −1, −1, −1) in flat space or can be generalized to the Schwarzchild metric in curved space. The Schwarzchild metric was one of the earliest relativistic metrics derived [8].

3.5 Minkowski

Minkowski developed the mathematical theory of spacetime, motivated by experimental observations requiring a new theoretical underpinning, but deriving this theory independently by means of a mathematical deduction. As Minkowski himself stated [10]:

Gentlemen! The conceptions of space and time which I would like to develop before you arise from the soil of experimental physics. Therein lies their strength. Their tendency is radical. From this hour on, space by itself and time by itself are to sink fully into the shadows and only a kind of union of the two should yet preserve autonomy. First of all I would like to indicate how, [starting] from the mechanics accepted at present, one could arrive through purely mathematical considerations at changed ideas about space and time.

Minkowski's framework for relativity was viewed by himself as geometrical theory regarding spacetime [10]. As a geometrical theory underlying relativity, based on judgments based on intuition, it would necessarily follow that relativity would be geo- metrical theory, and hence an analytic a priori theory in the Kantian terminology. Just as the geometry of relativistic spacetime runs counter to the common-sense intuition of Euclidean geometry, so do the consequences of this spacetime geometry.

A major consequence of the relativity theory is that time is relative, as exemplified by the phenomenon of time dilation. Is it clear, in fact, whether Kant believes in an absolute frame of reference in his notions of space and time? It is implied, for he would have subscribed to Euclidean geometry or Newtonian physics. Dirac points out the essential change introduced by relativity [11]:

What makes the theory of relativity so acceptable to physicists in spite of its going against the principle of simplicity is its great mathematical beauty. This is a quality which cannot be defined, any more than beauty in art can be defined, but which people who study mathematics usually have no difficulty in appreciating. The theory of relativity introduced mathematical beauty to an unprecedented extent into the description of Nature. The restricted theory changed our ideas of space and time in a way that may be summarized by stating that the group of transformations to which the spacetime continuum is subject must be changed from the Galilean group to the Lorentz group.

3.6 Euclidean Geometry in Kantian Philosophy

The empirical realism of Kant was the basis for his insistence on Euclidean geometry [12]. Kant believed that his appeal to a priori Euclidean constraints on space made possible his empirical geometry ... the three dimensionality of space ... is what makes it possible to demonstratively refer to unapprehended appearances that are included in the empirical content that the object of an outer empirical intuition must satisfy.

And according to Kant himself,We can properly only say, therefore, that, so far as we have hitherto observed, there is no exception to this or that rule.

3.7 Riemannian Geometry

The non-Euclidean geometry assumed by general relativity is Riemannian: spacetime itself has curvature, and it is a manifold in a higher dimensional space. The derivative of this geometry is arguably "non-intuitive," following from certain axioms to derive general properties of such a geometry. It introduces the notion of an n − −dimensional space, developed in the first half of the 19 th century by Riemann, who generalized the ideas in his

work [13]. On the one hand, it could be considered as analytic because it arises from geometry, on the other hand, arguably a synthetic a priori theory due to the deductions from previous work. This raises the question of whether the propositions of non-Euclidean geometry would be classified as analytic.

As Philip Kitcher [14] points out, regarding non-Euclidean properties known as S properties and the Reichenbachian space:

The upshot of this is that to recognize something as an S−property we already have to know what the properties of space are. Without knowing that we were not confronting the Reichenbachian space we could not take the angle-sum property to be an S property. The intuition is supposed, however, to show us that we are confronting Euclidean space. But we cannot draw this conclusion until we have distinguished the S-properties.

As according to Reichenbach, perception of the curved geometry is possible, thus thus the the Riemannian/Reichenbachian geometry is an analytic a priori theory. As a theory based on this geometry, arguably Einstein's relativity is an analytic a priori theory. The argument of the Riemannian geometry as an a priori intuition contradicts the interpretation of Cassirer in Substanzbegriff und Funktionsbegriff [15], which would tend to look upon space and time as intellectual forms and not a priori intuitions. 1

3.8 Particles in Physics: The Graviton

Einstein's theory relies heavily on the use of particles, such as photons, which mediate the electromagnetic interaction. Such objects seem to be the creation of deductions from cognition–we cannot sense them, nor do we have a knowledge of them from intuition, as space and time. They have certain physical and mathematical properties.

Yet this type of particle has an apodeictic quality, that their existence is necessary for the synthetical a priori judgments in spacetime. In other words, the laws of physics require the existence of a photon, which travels at the speed of light. The graviton, the particle which mediates the gravitational interaction, is likewise required by general relativity. Particles may have an effect on empirical observations, but they are things of pure understanding, divorced from any sensibility. They could be characterized as synthetical a priori. Insofar as they are objects of intuition, they could be characterized as analytic a priori.

The graviton was proposed as the mediator of the gravitational interaction. This particle was an arguably an a priori conception, when the graviton was only recently detected, nearly a century after being proposed [2]. In the Kantian paradigm, they are objects of pure understanding, which are conceived of a priori. They are separated from direct sensibility from our sense organs, but can be perceived as the result of the analysis of data. This particle may be "sensed" from detectors such as LIGO, but the detection is the result of pure understanding.

In addition, the relation between Einstein's relativity and Kantian a priori theory was the subject of a book by Riechenbach [16]. Reichenbach believed that Kant's theory had limited usefulness to relativity, although mentioning admiration for Kant. It is argued here that the notion of a priori theory remains useful despite Kant's description of a Newtonian system.

3.9 Fields and Geometry

General relativity could be seen as geometrizing the physical fields [17]. The distinguishing feature of general relativity was the inherent idea of the geometrization of the gravitational interaction, which marked a departure from traditional theories of physics [18].

The connection between general relativity and geometry is unmistakable. The field is merely a device for specifying the magnitude and direction of an interaction. In Einstein's general relativity, there is the field tensor which is abstracted beyond the electromagnetic field of special relativity. The general relativity fields obey the 4-dimensional Riemannian spacetime. However, there is a significant predictive power beyond geometry alone, as it is a unified field theory. Insofar as general relativity is a geometrical theory, the the theory would an analytic a priori theory in the Kantian categorization. As far as deriving from previous physical theories, it could be classified as a synthetic a priori theory. However, as much as Einstein promoted the role of geometry in theory of general relativity, Einstein didn't believe that General Relativity was a geometrization of physics [19]. Regardless, it is important to note the role of geometry.

4 Perspectives on Kant from Modern Philosophers

I contend that for Kant, in his characterization of science, the thought process used to arrive at a conclusion was paramount. This is what would separate an priori cognition from being analytic or synthetic. With respect to the theory of relativity, there is a dialectic as to its classification as analytic or synthetic, which depends in large part in the theory being based heavily on geometry, i.e., the spacetime geometry of Minkowski.

That Kant may have been referring to a Newtonian system, is the consequence of the age in which he lived· · · but his careful dissection of the thought process leading to a priori statements of science is of a more generalizable applicability. Kant's notions of space and time as intuitive would be seen in different perspective in the light of relativistic space time in which the distinctions between the three spatial dimensions and the time dimension are blurred as a consequence of the finite speed of light. The question of whether a conception is synthetic or analytic depends on the characterization of intuition. Clearly, deductions about spacetime, with respect to the geometry, are characterized in the Kantian system as intuitive deductions, since geometry is intuitive.

Hence conclusions based on geometry are analytic. But the question of geometry on a fourdimensional manifold being intuitive? Yes, in the sense that they are based on logical propositions. On the other hand, the development of the theory through thought experiments would indicate that the relativity would be characterized as synthetic a priori theory. From the Critique of Pure Reason, consideration of relativity according to the Transcendental Analytic and the Transcendental Logic yield to divergent conclusions about what kind of a priori theory we are dealing with: analytic in the former case, and synthetic in the latter case.

That we have demonstrated a seeming contradiction in Kant's reasoning of a priori judgments as applied to relativity can be explained by the work of another philosopher. Gottlob Frege [20], points out weaknesses in Kant's dichotomy between synthetic and analytic a priori judgments.

According to Frege's interpretation of the Kantian dichotomy: "our synthetic based on intuition cannot possibly be cut cleanly away from our analytic." He notes this in the case of a long mathematical proof, with steps based on intuition, but the total result of which is a work that is synthetic. We observed this problem earlier, where we discussed that synthetic judgments based on intuition, as in the case of special relativity, can- not be discerned from analytic judgments. Frege notes that Kant classifies arithmetic as synthetic a priori, but as it is based on logical propositions, isnt this classification deserving of fresh look? According to Frege, "The division of judgments into analytic and synthetic is not exhaustive." This would imply that there would be cases in which a judgement might not fall cleanly into one classification or the other. The theory of relativity is clearly a case where the nonexhaustiveness of this division is manifest.

Without sensibility no object would be given to us Frege argues, that we may have no sensibility of mathematical objects such as ∞. Kant is really discussing sensuous intuition, things that are in a sense tangible for the mind. In the case of relativity, for even a mathematical abstraction such as a photon, physicists have developed an intuition of mathematical rules.

Frege continues to emphasize the analytic nature of arithmetical propositions. Since arithmetical, or mathematical, propositions, can be derived from logical means, they are analytic. Theoretical physics, including special relativity, hence falls under this categorization, and hence, is analytic theory, whether or not it is based on geometry or not.

An analysis of Ludwig Wittgenstein's Philosophical Investigations implies that he treats seriously the value of synthetic a priori judgments [21]. Propositions of the form of empirical propositions, and not only propositions of logic, form the foundation of all operating with thoughts. Here, Wittgenstein implies that judgments could be arrived at from empirical propositions. Operating with thought, on empirical propositions, would be forming a synthesis in the Kantian terminology. Those empirical propositions which were certain would be classified in the Kantian system as a synthetic a priori judgments. Wittgenstein makes another remark useful for this discussion.
"Both propositions, the arithmetic one and the physical one, are on the same level. I want to say: the physical game is just as certain as the arithmetical. But this can be misunderstood. My remark is a logical and not a psychological one."

Therefore, we conclude, we may have propositions in physics which are a priori just like mathematical propositions which are a priori. Furthermore, since they are on the same level, we may have analytic propositions in physics just as in mathematics. So, it seems that Wittgenstein would follow the Kantian interpretation to a large extent, except admit the possibility of analytic a priori propositions in physics.

Wittgenstein asks a rhetorical question about physics which hints at the synthetic–analytic problem [22]. "Should I say 'I believe in physics', or 'I know that physics is true' ? This statement would tend to question whether physics is a priori, i.e. necessarily true, or whether synthetic and thus worthy of belief. He provides an example which demonstrates his synthetic a priori judgment of physics:

I am taught that under such circumstances this happens. It has been discovered by making the experiment a few times. Not that that would prove anything to us, if it weren't that this experience was surrounded by others which combine with it to form a system. Thus, people did not make experiments just about falling bodies but also about air Resistance and all sorts of other things. But in the end I rely on these experiences, or on the reports of them, I feel no scruples about ordering my own activities in accordance with them. - But hasn't this trust also proved itself? So far as I can judge- yes.

He has demonstrated that it is a synthesis of experiences, but also worthy of trust, and hence synthetic a priori. The significance of whether a theory such as relativity would be classified as analytic a priori or synthetic a priori is in understanding the fundamental significance of the theory to physics: if analytic a priori, then more fundamental to understanding of nature as it arises from the geometry of space and time itself.

5 Concluding Remarks

In concluding, we make some remarks on the relation between mathematics and theoretical physics, the simplification and aesthetic qualities of relativity, and the applicability of the discussion of Kantian philosophy and Einstein's relativity to unified field theories.

5.1 Relation between Mathematics and Physics

Whether analytic or synthetic a priori, modern physics is heavily rooted in mathematics. The uncanny relationship between mathematics was noted by physicist Eugene Wigner[23].

Mathematics does play, however, also a more sovereign role in physics. The laws of nature must have been formulated in the language of mathematics to be an object for the use of applied mathematics.

Geometry is fundamental to theoretical physics, as the latter describes the phenomena in a space time, whose properties are governed by mathematics. The greater the extent to which relativity is analytic, the more of a fundamental theory that it is. The greater the extent to which it is synthetic, it is an empirical and derivative theory. The resolution of the contradiction between analytic a priori and synthetic a priori is not peculiar to the case that we have studied. The dialectic quality that we have noted is not peculiar to general relativity. It is a general feature of field theory which respects certain symmetries. It may be argued that the quantum theory of fields, due to the geometrization of the physical fields, would have the same dilemma.

The fundamental applicability of mathematics to a physical description of nature has been noted by Dirac [11].
The physicist, in his study of natural phenomena, has two methods of mak- ing progress: (1) the method of experiment and observation, and (2) the method of mathematical reasoning. The former is just the collection of selected data; the latter enables one to infer results about experiments that have not been performed. There is no logical reason why the second method should be possible at all, but one has found in practice that it does work and meets with reasonable success. This must be ascribed to some mathematical quality in Nature, a quality which the casual observer of Nature would not suspect, but which nevertheless plays an important role in Nature's scheme.

5.2 Simplification and Abstraction in Relativity

It is worth noting the simplicity and elegance of general relativity. According to Dirac [11]:

The discovery of the theory of relativity made it necessary to modify the principle of simplicity. Presumably one of the fundamental laws of motion is the law of gravitation which, according to Newton, is represented by a very simple equation, but, according to Einstein, needs the development of an elaborate technique before its equation can even be written down. It is true that, from the standpoint of higher mathematics, one can give reasons in favor of the view that Einstein's law of gravitation is actually simpler than Newton's, but this involves assigning a rather subtle meaning to simplicity, which largely spoils the practical value of the principle of simplicity as an instrument of research into the foundations of physics.

Arguably, the simplicity of relativity arises from the analytic a priori nature, arising from naturally from geometric considerations, from Riemannian geometry, which was mentioned.

A level of abstraction of the theory of relativity is evident in the connection between space and matter, as described in the excerpt by Rovelli [24]. It's a moment of enlightenment. A momentous simplification of the world: space is no longer something distinct from matter it is one of the material components of the world. An entity that undulates, flexes, curves, twists. We are not contained within an invisible, rigid infrastructure: we are immersed in a gigantic, flexible snail shell.

The sun bends space around itself, and Earth does not turn around it because of a mysterious force but because it is racing directly in a space that inclines, like a marble that rolls in a funnel. There are no mysterious forces generated at the center of the funnel; it is the curved nature of the walls that causes the marble to roll. Planets circle around the sun, and things fall, because space curves.

To put relativity in context with the Newtonian view which pervades Kant's writings, abstraction was already necessary at the time of Isaac Newton. In invoking the action at a distance phenomenon, Newton describes [25]

That gravity should be ... so that one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed is so to me so great an absurdity ...

Newton continues in his writing that was missing was a perhaps even more fundamental explanation, but which was elusive to the scientists of his day. Metaphorically, if mathematics is a language, physics is a dialect thereof, subject to the constraint thatit reflect the properties of the universe. The different relations between the laws of physics would be manifest in a mathematical grammar, as is the case with relativity. The laws of physics must be mathematically valid: that is necessary but not sufficient condition. Theoretical physics begins with axioms, which are in the case of relativity, derived by recourse to intuition, albeit an intuition of geniuses such as Albert Einstein to comprehend the geometrical nature of Riemannian topological spaces. And so, it has been argued here that relativity should be classified as a analytic a priori system in the philosophical categorization put forth by Immanuel Kant.

5.3 Significance of Geometrization of Relativity and the Application to Unified Field Theories

To extend the argument, there are several consequences of the classification of relativity as an a priori theory. The significance of it is that it is then philosophically as well as physically a fundamental theory of the universe, as it is necessary to describe the universe. It may be argued that philosophy answers different questions about a theory than physics. However, it should be mentioned that it is not the theory of everything (TOE). For such a TOE would necessarily describe the quantum aspects of the universe. It should be mentioned that a truer appreciation of the universe has arisen from the quantum field theory (QFT) [26], with manifestations such as in electroweak unification [27] and the prediction and subsequent observation of the Higgs boson. Quantum gravity and string theory are examples of candidates for such a theory. The unification of electroweak with gravity might involve gravity-only large extra dimensions [28], such as occurring in Kaluza-Klein theories. Kant's philosophical legacy may be reflected in leaving room in physics for objects of intuition which weren't directly stated in the Critique of Pure Reason. The pursuit of unified field theory in historical context of relativity is described by Vizgin [18]:

Nevertheless, investigations of unified geometrized field theories continued. The most persistent adherent and leader of the program continued to be Einstein ... At the beginning of the 1920s, however, after the recent tri umph of general relativity, this direction appeared theoretically very deep and promising. At that time, many physicists regarded the geometrization program very highly and considered general relativity or Weyl's theory as the ideal of fundamental physical theory. In arguing that relativity is an analytic a priori theory, we can extend the argument to categorize unification theories. Let us focus attention on Kaluza-Klein theories, Once 16again, the element of Riemannian geometry is present in the topology of compact extra dimensions. Kaluza and Klein, while unifying only electromagnetism and gravity [29,

30], paved the way for theories which would unify all the fundamental forces. Arkani- Hamed described the theoretical argument for TeV-scale unification and sub-millimeter dimensions [31], relating the size of extra dimensions R and the fundamental Planck scale (M_P) to the effective Planck scale (M_D) in the extra dimensions, according to

$$\bar{M}_P = R^n M_D^{2+n} \qquad (5.1)$$

Due to the presence of the extra dimensions, the effective Planck mass is brought down to the TeV scale.

As Hawking states [32], "I shall describe how we are trying to find a unified theory that will include quantum mechanics, gravity, and all the other interactions of physics.

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If we achieve this, we shall truly understand the universe and our position in it." To understand place of mankind in the universe, is to understand truly the implications of a theory, which philosophy helps explain. To understand if a fundamental theory, such as relativity, or a TOE, is analytic a priori, helps physicists to appreciate the nature of the theory and how necessary it is to a description of the universe.

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