Calculation of Solid Fuel Rockets Optimal Values by Analytical Method

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Abstract

This article deals with the analytical method of solid fuel rocket calculating. Reference is made to methodology of determining mathematical functions, which have great importance for calculation not these rockets. In the article, as an example of use of analytical method, characteristic numbers dependencies on length of solid fuel grain and caliber of rockets (diameter of combustion chamber) are derived. Using these dependencies' (functions), and some analytical operations with them, it Is possible to determine main parameters of the rocket, and their optimal values..

Keywords: rocket, solid fuel, inner-ballistic process, exploitations, analytical method

Introduction to the analytical method of solid fuel rockets calculating

Name of the method is not derived from its "analytical purity", but from the fact that it is a guide, that shows how we can analytically advantageously solve some important problems of solid fuel rockets calculation. This analytical solution is clear and useful, especially if we consider existence of very powerful computing machines. Moreover, it has number of advantages, which includes:

1. It uses nowadays considerable experiences in design and implementation of solid fuel rockets, as well as previously existing methods of calculation.
2. It allows to mathematically express some significantly functional dependencies between parameters of solid fuel rockets and use them
   - to theoretically follow their reciprocal influence on each other,
   - to find optimal or another desired solution with great precision,
   - to make conclusions which have practical significance for design and exploitation.
3. It uses mechanical similarities between "model" (standard) and "product" of considered rocket.
4. It allows sufficiently plausible mathematical listing of inner-ballistic processes and determination of the most important parameters of rocket with respect to them.
5. It allows modifying or compiling mathematical relations favorable for solution on calculating machines.

Main objective and task of solid fuel rockets calculation method is to give it character of systematic, organized and purposeful activity that leads to clear results. At the same time it should be a guide to determination of main design, ballistic, exploitative and other parameters of designed, possibly adapted rocket. Analytical method also helps in solving one major, complex problem of solid fuel rockets calculation so called ballistic design and its optimization, thus calculation of the most favorable rocket parameters with respect to actual conditions of use.

The principle of this method consists of creation of substantiated simple functional dependencies between individual rocket parameters, joining them into more complex functions and use of these for complex computational problems solutions using analytical operations.

E.g.: Weight of bottom parts of solid fuel rockets can be written as a function of rocket caliber as follows:

$$m_{DN} = K_{DN}D^3$$

Weight of combustion chamber can be expressed as a function of caliber and grain length as:

$$m_{SK} = K_{SK}D^2L$$

where: K_{DN} – coefficient of bottom parts,
D – rocket caliber,
K_{SK} – coefficient of combustion chamber,
L – lenght of solid fuel grain.

Based on these and similar relations, we canwrite speed (Tsiolkovsky´s) numbers as function of caliber and length of a grain as follows

$$c = 1 + \frac{\beta_1 D^2 L}{\beta_3 + \beta_4 D^3 + \beta_S D L^2}$$

If we partially derive this function and solve condition

$\frac{\delta_c}{\delta_L} = 0,$

we get the expression which determines optimal length of a grain as a function of caliber

$$\boldsymbol{L_{OPT}} = \frac{-\boldsymbol{\beta_2}\left(\boldsymbol{\beta_3} + \boldsymbol{\beta_4 D^3}\right) + \sqrt{\boldsymbol{\beta_2^2}(\boldsymbol{\beta_3} + \boldsymbol{\beta_4 D^3})^2 + \boldsymbol{\beta_1\beta_2\beta_5 D^3}(\boldsymbol{\beta_3} + \boldsymbol{\beta_4 D^3})}}{\boldsymbol{\beta_2 \rho'_5 D^3}}$$

At this grain length, rocket of any caliber D > 0, reaches maximal speed numbers. Thus, we determine one of the most important parameters of solid fuel rockets - optimal grain length with respect to the speed number. β1, β2, β3, β4 and β5 are constants, and will be derived further. Entire expression can then be adjusted and considerably simplified.

Use of analytical method for deriving functional dependencies of characteristic numbers on rocket caliber and length of solid fuel grain.

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It is generally well known that during calculation and construction of rockets, we often monitor values of characteristic numbers, which show us rocket design perfection, very closely. Significance of characteristic numbers is often discussed in variety of professional and popular literature. Many foreign experts, such as prof. Staňukovič, Dutch engineer Verteght and others are dealing with the task of calculating rockets with respect to characteristic numbers, in great detail and certainly not without reason. J.A. Pobedonoscev in publication "Earth satellite" and a number of journal articles and books authors, domestic as well as foreign, point on specific examples, how big economic importance has calculation of rocket, especially multistage one, if it complies with principles of calculation with respect to characteristic numbers and their optimal values. Previously known theories about characteristic numbers all primarily address only question of determining their absolute values and are based on the assumption that design allows to achieve these values. Initial terms and conditions must be relatively highly accurate, which requires considerable insight and experience of designer on rockets similar as counted rocket. Such calculation is therefore highly influenced by subject, which is undesirable.

We will show you how is possible to reduce influence of subjective factor on case of using analytical method for determining functional dependencies of characteristic numbers to main parameters of rocket, which are caliber and length of solid fuel grain.

Let us, debate; mention conditions and prerequisites that must be met, so that we can advantageously use abovementioned method:

1. Weight of payload must be known.
2. Control apparatus weight determination must be possible.
3. Choose a suitable solid fuel with known combustion law.
4. Choose sure way of rocket motor elaboration (solid fuel tubular grains, grain with countersink internal combustion, etc.).
5. Determine construction scheme (conception) of rocket (backswept, rotating, etc.).
6. Select construction material of main rocket parts.

For demonstration of used analytical method, we subdivide processed issue into three parts, corresponding with principle of method:

I. Weights of rocket parts as a function of caliber and grain length.
II. Characteristic numbers as functions of caliber and grain length.
III. Determination of rocket basic parameters by derived functions.

1. Weights of rocket parts as a function of caliber and grain length.

Let´s have backs wept rocket, which general constructional scheme is shown in Fig. 1.

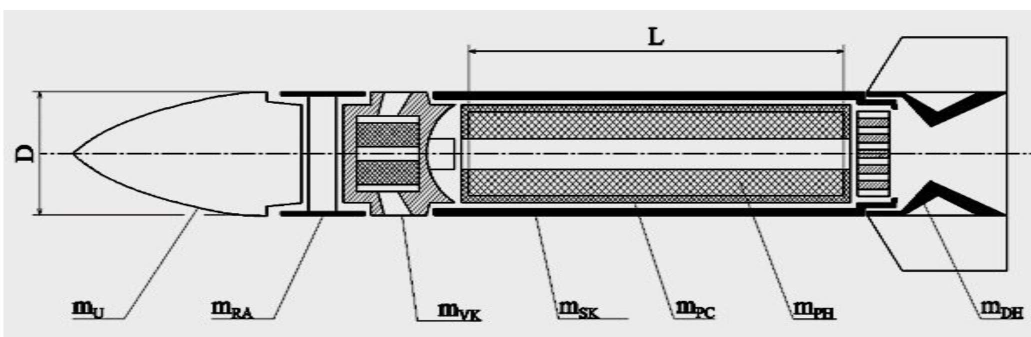

Fig. 1: General constructional scheme of backswept solid fuel rocket.
Meaning of used marks:

m_U – weight of payload,
m_{RA} – weight of control apparatus,
m_{VK} – weight of cover parts,
m_{SK} – weight of combustion chamber,
m_{PC} – weight of armoring,
m_{PH} – weight of solid fuel filling,
m_{DN} – weight of bottom parts.

Note: armoring

Weight of cover parts often includes: weight of ignition element, cover itself, event. rocket motor for additional rotation.

Weight of bottom parts includes: weight of stabilization, jets (spray-nozzle bottom) and possible grid.

Suppose that listed constructional scheme of rocket motor is elaborated by unspecified solid fuel grain with star inner burning, as seen in Fig. 2

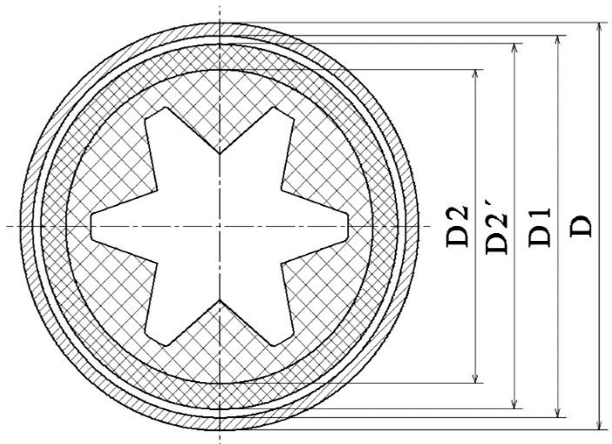

Fig. 2 Cross-section of rocket motor

Meaning of used marks:
D – outer diameter of the combustion chamber(identical with caliber),
D_1 – inner diameter of combustion chamber,
D'_2 – outer diameter of armoring,
D_2 – outer diameter of solid fuel grain (identical with inner diameter of armoring).

1. *Weight of payload(m_U)*

According to above mentioned assumptions applies:

$$m_U = const. \quad (1)$$

2. *Weight of cover parts (m_{VK})*

Is very often given, especially for small rockets, by theoretically justifiable relation

$$m_{VK} = K_{VK} D^3 \quad (2)$$

Practical calculations however show, that in certain circumstance, particularly for larger rockets applies

$$m_{VK} = K_{VK} D \quad (3)$$

where K_{VK} and K_{VK}'are coefficients of cover parts.

3. *Weight of bottom parts(m_{DN})*

Is determined very often by

$$m_{DN} = K_{DN} D^3 \quad (4)$$

Practical calculations show that in some cases, particularly for larger backswept rockets, is more convenient to use relation

$$m_{DN} = K_{DN}^{'} D \qquad (5)$$

where K_{DN} and $K_{DN}^{'}$ are coefficients of bottom parts.

4. Weight of control apparatus (m_{RA})

After corresponding analysis and choice of control apparatus for given type of rocket, its weight can be considered for constant, i.e.

$$m_{RA} = const. \qquad (6)$$

5. Weight of combustion chamber (m_{SK})

On Fig. 1and2, we see that weight of combustion chamber is determined by relation

$$m_{SK} = \frac{\pi}{4}(D^2 - D_1^2)\rho_K L \qquad (7)$$

where ρ_K- specific weight of combustion chamber material.
Inner diameter of combustion chamber is given by relation

$$D_t = K_1 D_D \qquad (8)$$

whereby

$$K_1 = \frac{2\sigma_K}{2\sigma_K + \sqrt{3\rho_K}}$$

where σ_K – allowable stress of combustion chamber material,
ρ_K – constructional pressure in combustion chamber.

If we substitute equation (8) into (7), after modifications, we get

$$m_{SK} = K_{SK} D^2 L \qquad (9)$$

where

$$K_{SK} = \frac{\pi}{4}\rho_K(\quad)1 - K_1^2$$

6. Weight of armoring (m_{PC})

On Fig. 1and2, we see that weight of armoring is given by

$$m_{PC} = \frac{\pi}{4}\left(D_2^{'2} - D_2^2\right)\rho_P L \qquad (10)$$

where ρ_P- specific weight of armoring.

Outer diameter of armoring can be expressed

$$D_2^{'} = K_2^{'} D,$$

where

$$K_2^{'} = K_1 - 2K_m \qquad (11)$$

K_ω is coefficient of expansion gap between armoring and combustion chamber wall. It depends on expected rocket caliber and dilatation properties of solid fuel, armor and combustion chamber.
Outer diameter of solid fuel grain can be expressed by relation

$$D_2 = K_2 D \qquad (12)$$

where
$K_2 = K_2' - 2K_P$.

K_P is coefficient of armoring thickness. Its value depends on dilatation characteristics of solid fuel and armoring, but also from heat-protective properties of armoring and anticipated rocket caliber.
If we substitute equations (11) and (12) to (10), after modification we get

$$m_{PC} = K_{PC} D^3 L \qquad (13)$$

where

$$K_{PC} = \frac{\pi}{4}\rho_P\left(K_2'^2 - K_2^2\right)$$

7. *Weight of solid fuel filling (m_{PH})*

In consent with Fig.2is given by relation
$m_{PH} = (A_{EF} - A_V)\rho_{PH} L$ (14)

Effective cross-section of combustion chamber(A_{EF}) can be expressed by relation

$$A_{EF} = K_{EF} D^2 \qquad (15)$$

where

$$K_{EF} = \frac{\pi}{4} K_2^2$$

Free cross-section (A_V) for neutral star internal combustion is given by

$$A_V = \frac{C}{Z} K_{SV} D L \qquad (16)$$

where C – throttling,
Z – stoppage,
ρ_{PH} – specific weight of solid fuel.

Grain shape constant K_{SV} is in this case calculated according to

$$K_{SV} = nK_2 \frac{\cos\frac{\theta}{2}}{\sin\varepsilon\frac{\pi}{n} + \cos\frac{\theta}{2}}\left[\frac{\sin\varepsilon\frac{\pi}{n}}{\sin\frac{\theta}{2}} + \frac{\pi}{n}(1-\varepsilon)\right]$$

where n – number of star points,
ε – coefficient of channel (for provisional calculations, it is appropriate to choose value ε =0,7),
Θ – corner angle, guaranteeing neutral combustion. (Its values are given e.g. in the book „Rocket propulsion“ by Barrére *et al.*).

If we substitute expressions (15) and (16) to (14), after modification we get

$$m_{PH} = \alpha_1 D^2 L - \alpha_2 D L^2 \tag{17}$$

where $\alpha_1 = K_{EF}\rho_{PH,}$

$$\alpha_2 = \frac{C}{Z}\rho_{PH}K_{SV}$$

I. Characteristic numbers as a function of caliber and grain length

Using derived relations for expressing weight of individual rocket parts, we can easily get final expressions, according to which, characteristic numbers vary depending on caliber and length of solid fuel grain.

1. Constructional number as a function of caliber and grain length

Constructional number can be written in expanded form

$$S = 1 + \frac{m_{PH}}{m_{RA}+m_{VK}+m_{DN}+m_{PC}+m_{SK}} \tag{18}$$

If we substitute required derived relations to equation (18), after modification, we get

$$S = 1 + \frac{\alpha_1 D^3 L - \alpha_2 D L^2}{\alpha_3+\alpha_4 D^3+\alpha_5 D^2 L} \tag{19}$$

where$\alpha_3 = m_{RA}$,
$\alpha_4 = K_{VK} + K_{DN}$,
$\alpha_5 = K_{SK} + K_{PC}$,

or

$$S = 1 + \frac{\alpha_1 D^2 L - \alpha_2 D L^2}{\alpha_3+\alpha_4'+\alpha_5 D^5 L} \tag{20}$$

where$\alpha_4' = K_{VK}' + K_{DN}'$

2. Speed (Tsiolkovsky´s) number as a function of caliber and grain length

Speed number can be written in expanded form

$$c = 1 + \frac{m_{PH}}{m_U+m_{RA}+m_{VK}+m_{DN}+m_{PC}+m_{SK}} \tag{21}$$

If we substitute required relations to equation (21) after modification, we get

$$c = 1 + \frac{\beta_1 D^3 L - \beta_2 D L^2}{\beta_3+\beta_4 D^3+\beta_5 D^2 L} \tag{22}$$

where $\beta_1 = \alpha_1$, $\beta_2 = \alpha 2$, $\beta 5 = \alpha 5$, $\beta_4 = \alpha 4$, ale $\beta_3 = m_U + m_{RA}$,

similarly

$$c = 1 + \frac{\beta_1 D^2 L - \beta_2 D L^2}{\beta_3 + \beta_4' D + \beta_5 D^2 L} \quad (23)$$

where $\beta_4' = \alpha_4' = K_{VK}' + K_{DN}'$

3. *Traffic (weight) number as a function of caliber and grain length*

Traffic number can be written in expanded form

$$p = 1 + \frac{m_{RA} + m_{VK} + m_{DN} + m_{SK} + m_{PC} + m_{PH}}{m_U} \quad (24)$$

If we substitute required derived relations to equation (24) after modification, we get

$$p = \gamma_1 - \gamma_2 D L^2 + \gamma_3 D^2 L + \gamma_4 D^3 \quad (25)$$

where

$$\gamma_1 = 1 + \frac{m_{RA}}{m_U}$$

$$\gamma_2 = \frac{C}{Z} \frac{\rho_{PH}}{m_U} K_{SV}$$

$$\gamma_3 = \frac{K_{SK} + K_{PC} + K_{EF}\theta_{PH}}{m_U}$$

$$\gamma_4 = \frac{K_{VK}' + K_{DN}'}{m_U}$$

similarly

$$p = \gamma_1 - \gamma_2 D L^2 + \gamma_3 D^2 L + \gamma_4' D$$

Where

$$\gamma' = \frac{K_{VK}' + K_{DN}'}{m_U} \quad (26)$$

II. Determination of rocket basic parameters by derived functions

We will be limited there, for brevity, to certain special cases of determination of solid fuel rocket parameters using some higher to derived relations, which have the greatest practical importance.

1. *Optimal grain length in terms of speed number*

Generally, we get it by solving condition $\frac{\delta_c}{\delta_L} = 0$.

Provided that, for c = f (L, D) we use equations (23), we get

$$L_{opt} = \frac{-\beta_2(\beta_3+\beta'_4 D)+\sqrt{\beta_2^2(\beta_3+\beta'_4 D)^2+\beta_1\beta_3\beta_5 D^3(\beta_3+\beta'_4 D)}}{\beta_2\beta_5 D^2} \qquad (27)$$

With this grain length, rocket of any caliber D > 0 reaches maximal speed number. This can be mathematically proven as summing D = const., when we get function c = f (L), which has extreme maximum.

2. *Rocket caliber necessary to achieve desired speed number.*

This task is practically significant. For its solution, we substitute expression (27)to equation (23), by which we get speed number dependency on rocker caliber, ergo function c = f (D). There is also already included condition of optimal grain length, which is advantageous in terms of clarity of the result. However, this function c = f (D) can be difficult after algebraic modifications. It is therefore appropriate, to tabulate it for practical calculation. Then we will calculate corresponding values of speed number for chosen calibers, by which we acquire course of function = f (D).

We set rocket caliber necessary to attain desired speed number by interpolation or extrapolation or from diagram of function c=f(D), constructed on the basis of already counted table.

3. *Maximal weight of solid fuel filling*

In case we want to detect basic dimension parameters of rocket, corresponding to maximum weight of solid fuel filling, we use equation (17). We must satisfy only with specified length of a grain when there is the most extreme maximum, as in terms of caliber is function monotonically progressive. Therefore it is sufficient to solve condition $\frac{dm_{PH}}{dL} = 0$

from which we get after modification

$$L_m = \frac{K_{EF}Z}{2K_{SV}C}D \qquad (28)$$

where L_m is grain length corresponding to maximal solid fuel weight for rocket of any caliber, where inner-ballistic conditions for proper rocket motor operation are respected.

Conclusion

This article describes analytical method of calculating of rocket motor which works on solid fuel. There is description of methodology for determination of mathematical functions that are important for calculation of these rockets. In our article is analytical method listed as example, dependencies of characteristic numbers of cartridge length, solid fuel and rocket body diameter is derived. With help of these dependencies (functions), we are able to determine main parameters of rocket and their optimal values in some analytical operations.

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